

# Differential Privacy: What, Why and When

## A Tutorial

**Moni Naor מוני נאור**

Weizmann Institute of Science



Slides credit: Guy Rothblum,  
Kobbi Nissim, Cynthia Dwork...

# What is Differential Privacy?

- Differential Privacy is a concept
  - Motivation
  - Rigorous mathematical definition
  - Properties
  - A measurable quantity
- Set of algorithmic techniques for achieving it
- First defined in:
  - *Dwork, McSherry, Nissim, and Smith*, **Calibrating Noise to Sensitivity in Private Data Analysis**, Third Theory of Cryptography Conference, TCC 2006.
  - Earlier roots: *Warner*, **Randomized Response**, 1965

# Why Differential Privacy?

- DP: Strong, **quantifiable**, **composable** mathematical privacy **guarantee**
- **Provably resilient** to **known** and **unknown** attack modes!
- Theoretically: DP enables many computations with personal data while preserving personal privacy
  - Practicality in first stages of validation

Not a panacea

# Good References

- **The Algorithmic Foundations of Differential Privacy**

Cynthia Dwork and Aaron Roth

<http://www.cis.upenn.edu/~aaroht/privacybook.html>

- **The Complexity of Differential Privacy**, Salil Vadhan

- **Differential Privacy: A Primer for a Non-technical Audience**

[https://privacytools.seas.harvard.edu/files/privacytools/files/pedagogical-document-dp\\_new.pdf](https://privacytools.seas.harvard.edu/files/privacytools/files/pedagogical-document-dp_new.pdf)

# Privacy-Preserving Analysis: The Problem



- Given dataset with **sensitive personal info** Health, social n/w, location, communication,
- How to compute and release **functions of the dataset**
- While protecting **individual privacy** Academic research, informed policy, national security

# Glorious Failures of Traditional Approaches to Data Privacy

- Re-identification [Sweeney '00, ...]
- Auditors [Kenthapadi, Mishra, Nissim '05]
- Genome-Wide association studies (**GWAS**) [Homer et al. '08]
- Netflix Prize [Narayanan, Shmatikov '08]
- Social networks [Backstrom, Dwork, Kleinberg '11]
- Attack on statistical aggregates [Dwork, Smith, Steinke, Ullman Vadhan '15]

# The Netflix Prize

- **Netflix Recommends Movies to its Subscribers**
  - Seek an improved recommendation system
  - Offered \$1,000,000 for “10% improvement”
  - Published training data

Prize won in September 2009  
“BellKor's Pragmatic Chaos team”

Very influential competition  
in machine learning
















# From the Netflix Prize Rules Page...

- “The training data set consists of more than 100 million ratings from over 480 thousand randomly-chosen, **anonymous** customers on nearly 18 thousand movie titles.”
- “The ratings are on a scale from 1 to 5 (integral) stars. **To protect customer privacy, all *personal information identifying* individual customers has been removed and all customer ids have been replaced by randomly-assigned ids.** The **date** of each rating and the title and year of release for each movie are provided.”



# Netflix Data Release [Narayanan-Shmatikov 2008]

- Ratings for subset of movies and users
- Usernames replaced with random IDs
- Some additional perturbation

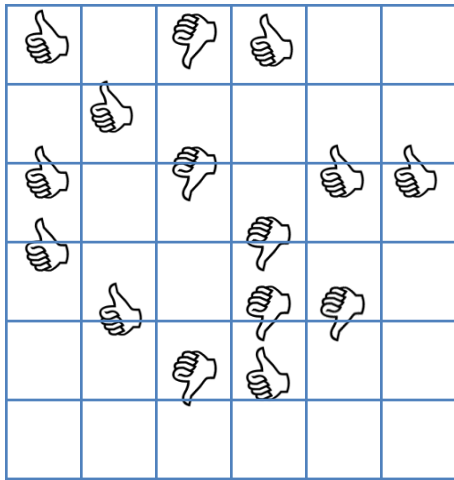
	Item 1	Item 2			Item M	
User 1						
User 2						
						
						
						
User N						

# A Source of Auxiliary Information

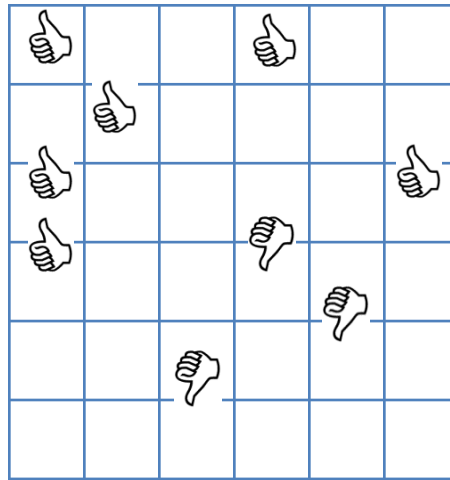
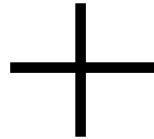


- Internet Movie Database (IMDb)
  - Individuals may register for an account and rate movies
  - **Need not be anonymous**
    - **Probably want to create some web presence**
  - Visible material includes ratings, **dates**, comments

# Use Public Reviews from IMDb.com

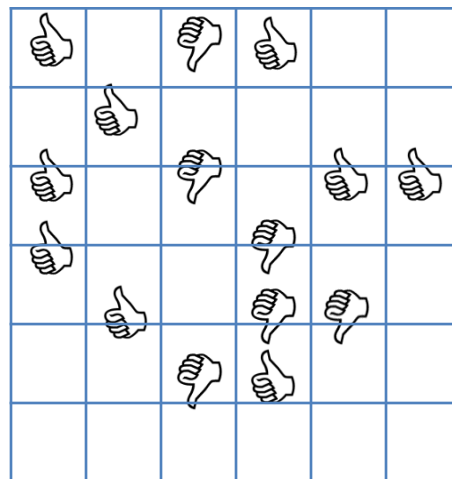


**Anonymized**  
Netflix data



Public, incomplete  
**IMDB** data

Alice  
Bob  
Charlie  
Danielle  
Erica  
Frank



**Identified** Netflix Data

# De-anonymizing the Netflix Dataset

## Results

of which 2 may be completely wrong

- “With 8 movie ratings and **dates** that may have a 3-day error, 96% of Netflix subscribers whose records have been released can be uniquely identified in the dataset.”
- “For 89%, 2 ratings and **dates** are enough to reduce the set of plausible records to 8 out of almost 500,000, which can then be inspected by a human for further deanonymization.”

## Consequences?

Settled, March 2010

- Learn about movies that IMDB users didn't want to tell the world about...

Sexual orientation, religious beliefs

- **Subject of lawsuits**

US Video Privacy  
Protection Act 1988

# Perfect Privacy?

Why not “**Semantic Security**”?

[a la Goldwasser Micali]

*Anything* that can be learned about a participant **from sanitized data**, can be **learned without it**

[Dalenius77]

**Unachievable:** **Auxiliary information** is a problem

[Dwork Naor]

Common theme in privacy horror stories

# A “New” Approach to Privacy

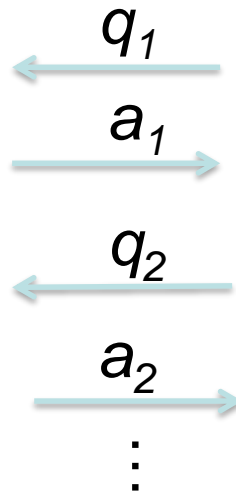
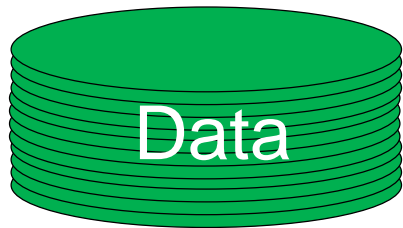
## Differential Privacy [DMNS06]

Any outcome is **equally likely** when I’m  
**in the database** or **out of the database**

Risk incurred **by participation** is low



# Learning Can Hurt

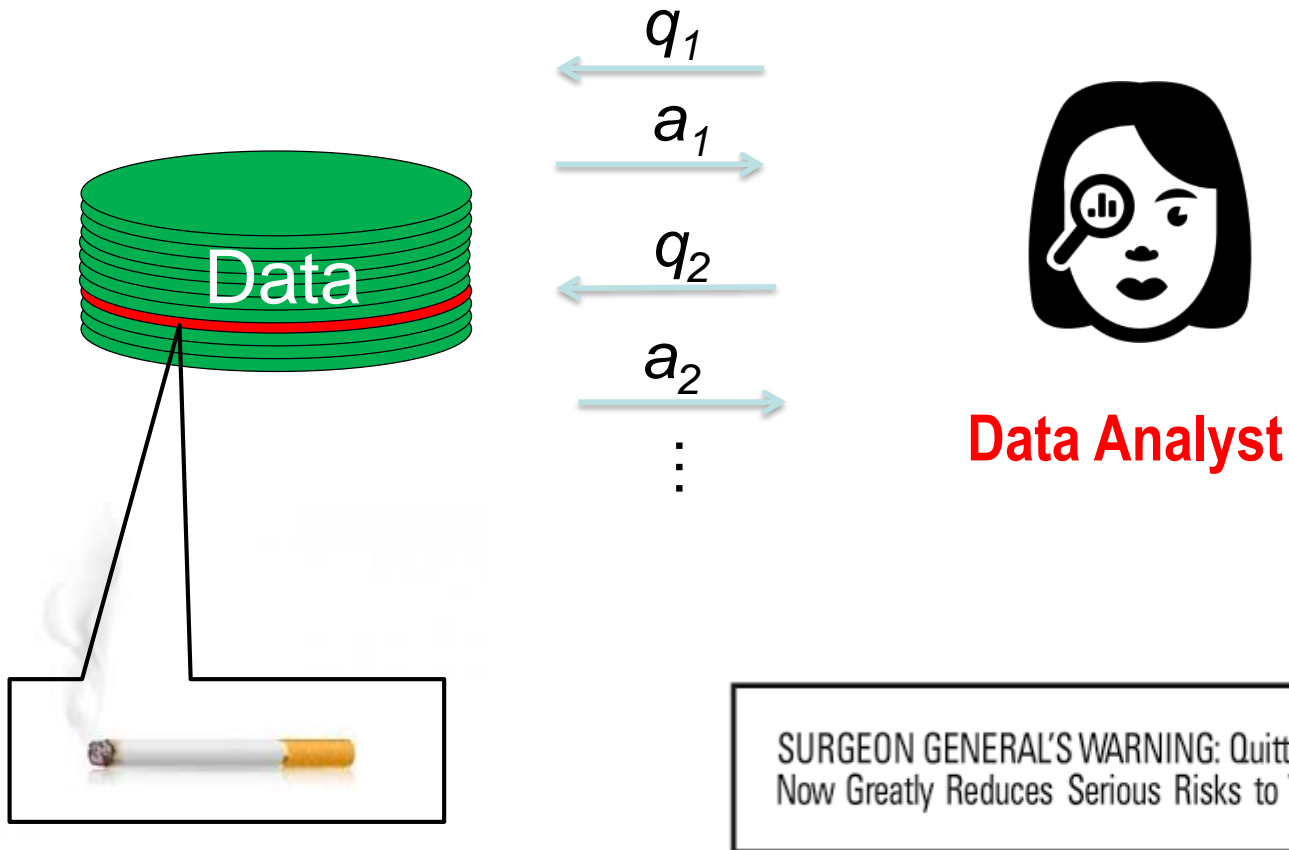


**Data Analyst**



SURGEON GENERAL'S WARNING: Quitting Smoking  
Now Greatly Reduces Serious Risks to Your Health.

# Teachings vs. Participation





# Differential Privacy

Any outcome is equally likely when I'm  
in the database or out of the database

Algorithm  $A$  guarantees  $\epsilon$ -differential privacy

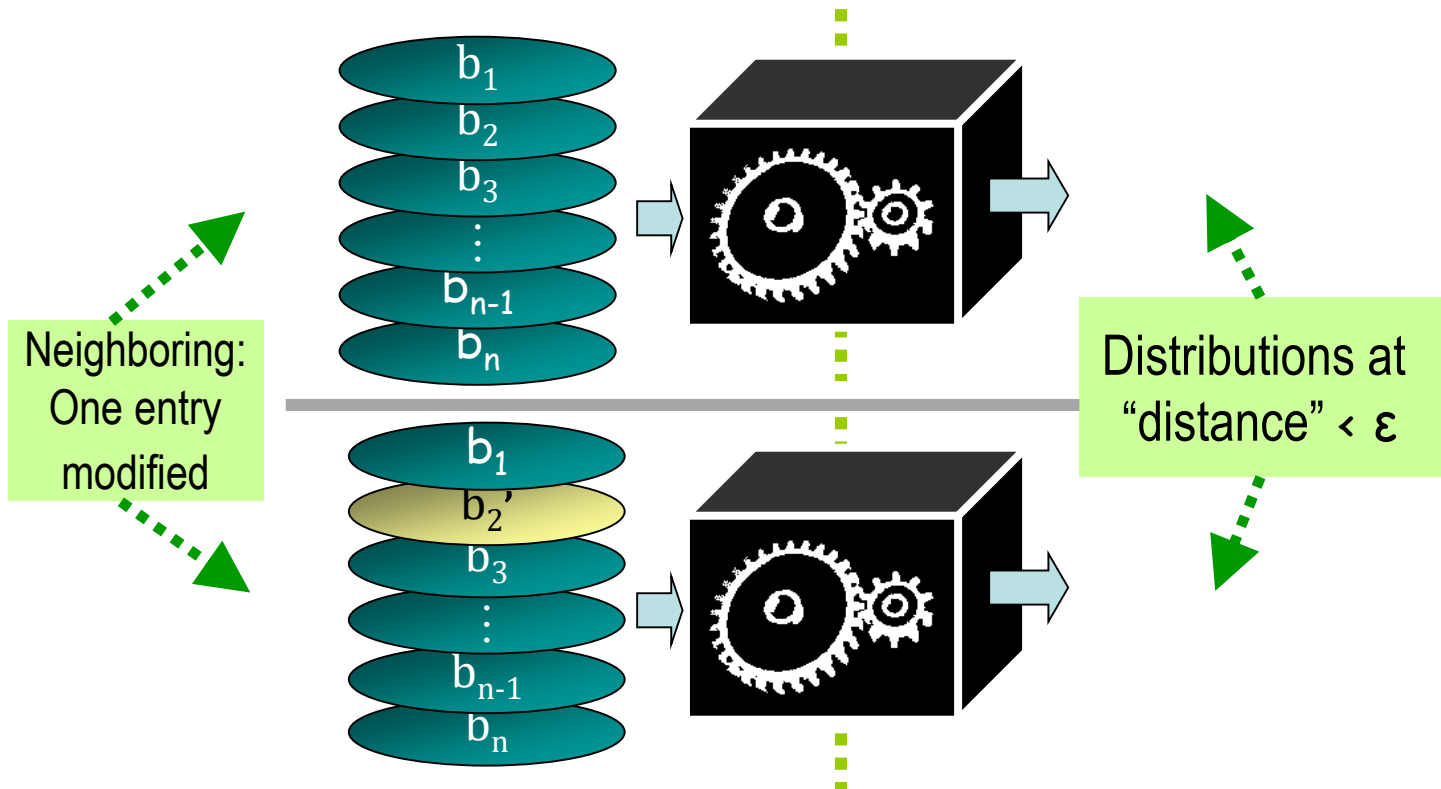
if for all DBs  $D$  and all events  $S$ :

$$Pr_A[A(D + me) \in S] \leq e^\epsilon \cdot Pr_A[A(D - me) \in S]$$


$$1 + \epsilon$$

Randomness introduced by  $A$

# Differential Privacy



# Differential Privacy

Any outcome is equally likely when I'm  
in the database or out of the database

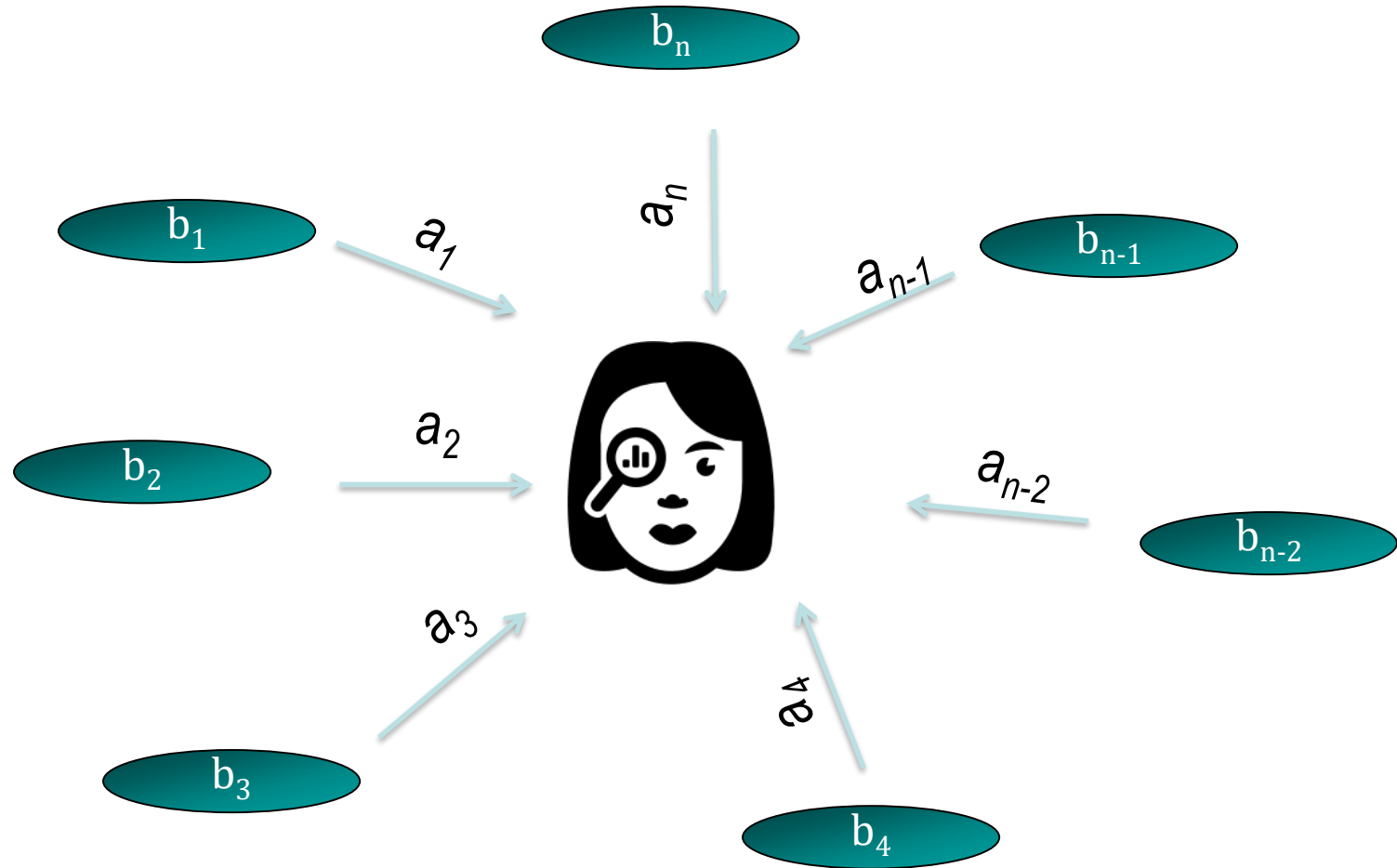
Algorithm  $A$  guarantees  $\epsilon$ -differential privacy

if for all DBs  $D$  and all events  $S$ :

$$Pr_A[A(D + me) \in S] \leq e^\epsilon \cdot Pr_A[A(D - me) \in S] + \delta$$

Randomness introduced by  $A$   $(\epsilon, \delta)$ -differential privacy

# Local Model



# Differential Privacy is a Success

- Algorithms in many setting and for many tasks

Important Properties:

Programmable!

- **Group privacy**:  $\epsilon k$  privacy for a group of size  $k$
- **Composability**
  - Applying the sanitization several time: graceful degradation
  - proportional to number of applications
  - **even prop. to squareroot of number of applications.**
- Robustness to **side information**
  - No need to specify **exactly** what the adversary knows
  - Postprocessing

Hard to quantify

# Differential Privacy: A Tutorial

- Basic composition

Answering **small** numbers of queries

- Advanced composition

Answering **moderate** numbers of queries

- Coordinated mechanisms

Answering **huge** number of queries

- Example of Mixing MPC and DP for passwords

# Composition

Privacy maintained even under **multiple analyses**

## Core issue

The key to differential privacy's success!

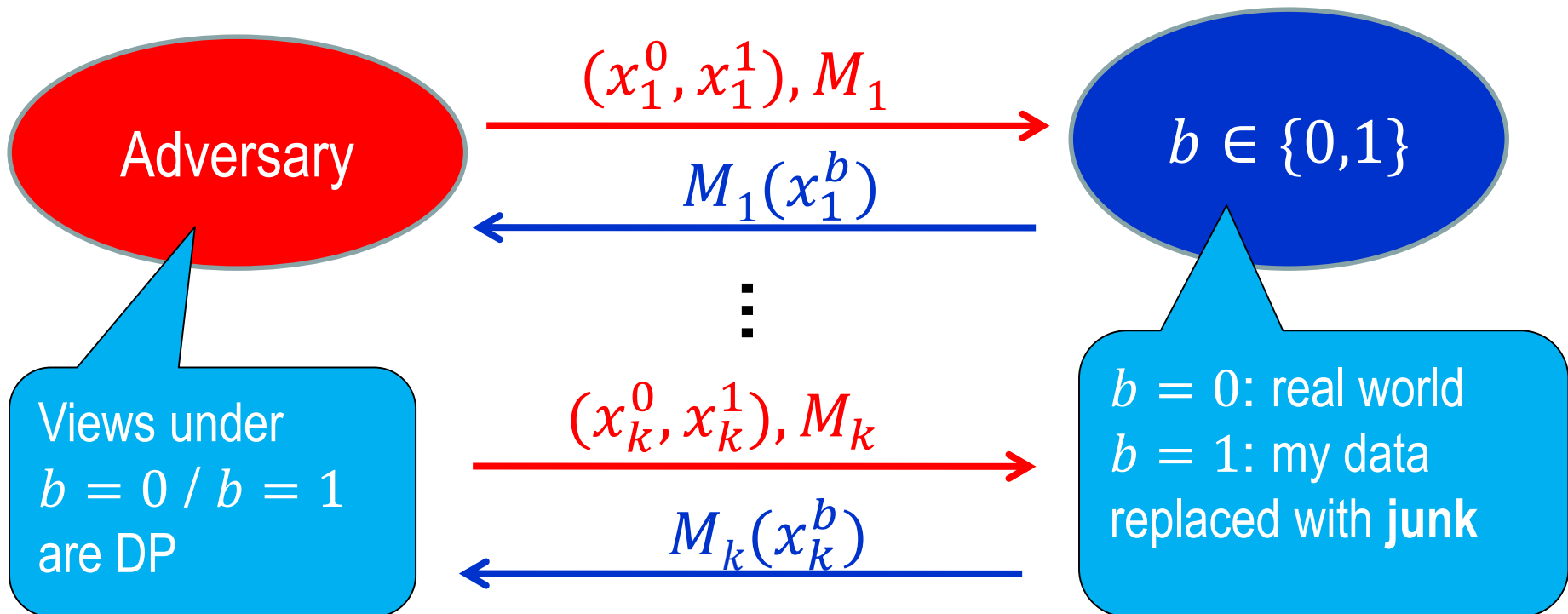
- Unavoidable
  - In reality, there are multiple analyses
- Makes DP “programmable”
  - Private subroutines make for private algorithms

# Composition

Privacy maintained even under **multiple analyses**

How do we define it? [DworkRothblumVadhan10]

- Adaptive, adversarial DBs and algorithms





# Basic Composition

- $k$  (adaptively chosen) algorithms, each  $\varepsilon_0$ -DP:  
taken together still  $k \cdot \varepsilon_0$ -DP

Application: answering multiple queries

# Basic Composition Proof

Define:  $M_{1,2}(x) = (M_1(x), M_2(x))$

$$\frac{\Pr[M_{1,2}(x)=(z_1,z_2)]}{\Pr[M_{1,2}(y)=(z_1,z_2)]} = \frac{\Pr[M_1(x)=z_1]\Pr[M_2(x)=z_2]}{\Pr[M_1(y)=z_1]\Pr[M_2(y)=z_2]} \leq e^{\varepsilon_1} e^{\varepsilon_2}$$

Property of the definition

- Independent of the implementation
- What about the **adaptive case?**

# Statistical queries

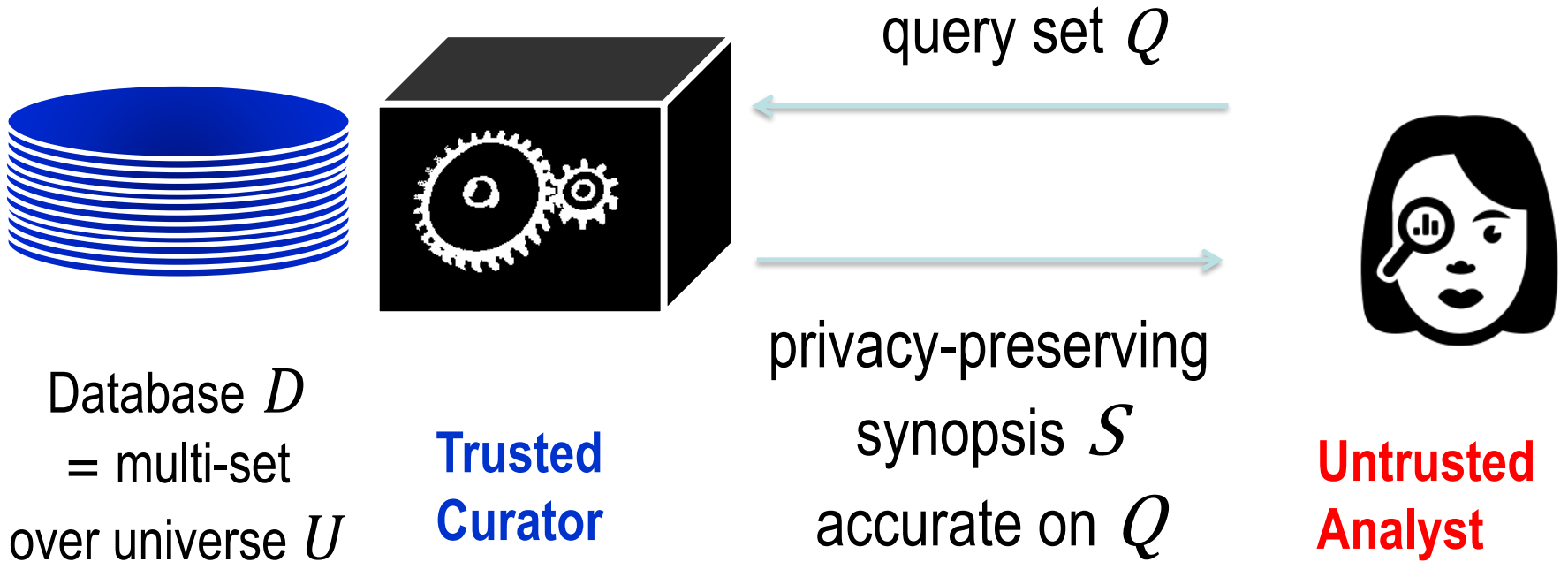
$q(D) =$  “*how many in  $D$  satisfy predicate  $P$ ?*”

$P$  is a Boolean predicate on universe  $U$

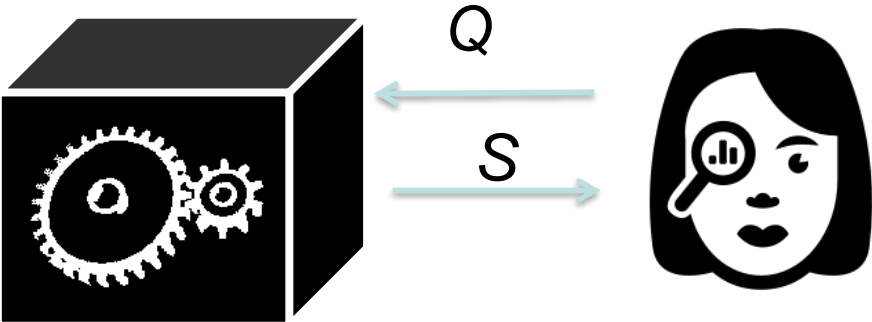
**statistical queries allow powerful data analyses**

- Perceptron, ID3 decision trees, PCA/SVM, k-means [BlumDworkMcSherryNissim05]
- any SQ-learning algorithm [Kearns98]
  - includes “most” known PAC-learning algorithms

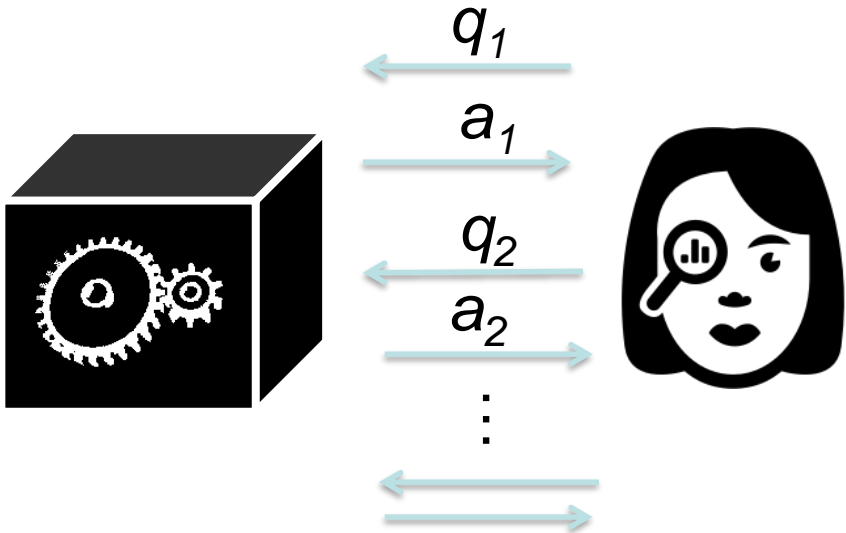
# Data Analysis Model



Offline: non-interactive



Online: interactive



# Answering a single counting query

$U$  is set of tuples:  $(name, tag \in \{0,1\})$

Counting query: # of participants with  $tag = 1$

**A:** output # of 1's + noise

**Differentially private!** For proper noise

Choose noise from **Laplace distribution**

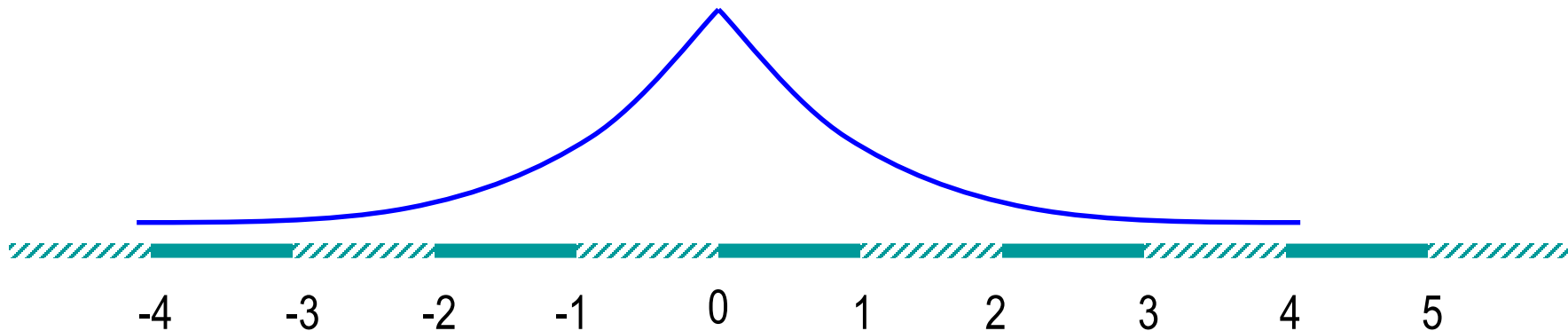
# Laplacian Noise

Laplace distribution  $Y = \text{Lap}(b)$  density function

$$\Pr[Y = y] = \frac{1}{2b} e^{-|y|/b}$$

Standard deviation:  $O(b)$

Set  $b = 1/\varepsilon$ , get that  $\Pr[Y = y] \propto e^{-\varepsilon \cdot |y|}$



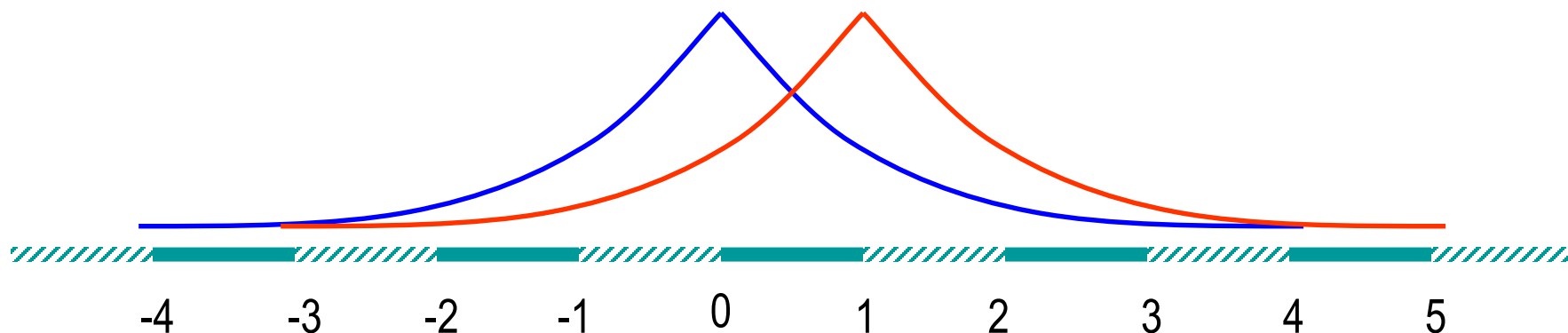
# Laplacian Noise: $\epsilon$ -Privacy

Take  $b = 1/\epsilon$ , get that  $\Pr[Y = y] \propto e^{-\epsilon \cdot |y|}$

Release:  $q(D) + \text{Lap}(1/\epsilon)$

For adjacent  $D, D'$ :  $|q(D) - q(D')| \leq 1$

For any  $z$ :  $e^{-\epsilon} \leq \Pr_{by D}[z] / \Pr_{by D'}[z] \leq e^{\epsilon}$



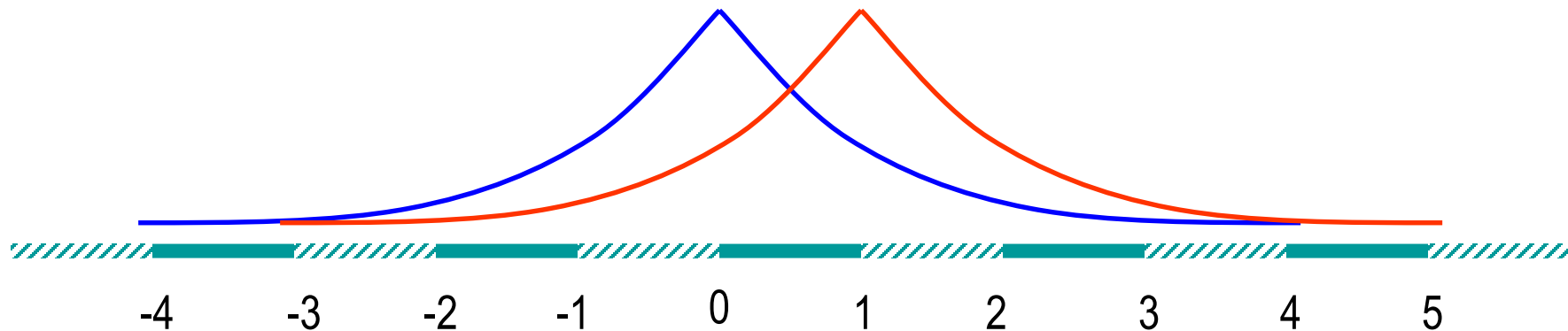


# Laplacian Noise: $\tilde{O}(1/\varepsilon)$ -Error

Take  $b = 1/\varepsilon$ , get that  $\Pr[Y = y] \propto e^{-\varepsilon \cdot |y|}$

$$\Pr_{y \sim Y}[|y| > k \cdot 1/\varepsilon] = O(e^{-k})$$

Expected error is  $1/\varepsilon$ , w.h.p error is  $\tilde{O}(1/\varepsilon)$



# Scaling Noise to Sensitivity [DMNS06]

Global **sensitivity** of query  $q: Un \rightarrow [0, n]$

$$GS_q = \max_{D, D'} |q(D) - q(D')|$$

For a counting query  $q: GS_q = 1$

Previous argument generalizes:

For query  $q$ , release  $q(D) + Lap(GS_q/\epsilon)$

- $\epsilon$ -private
- error  $\tilde{O}(GS_q/\epsilon)$

# Answering $k$ Queries: Basic Composition

Answer  $k$  queries, each with sensitivity 1

- Use Laplace with  $\epsilon_0 = \epsilon/k$  **privacy per query**

Better privacy, more noise per query ( $\sim \text{Lap}(k/\epsilon)$ )

- **Composition**:  $\epsilon$ -privacy for all  $k$  answers

Error (roughly) **linear in number of queries**

- E.g.: can answer  $\sqrt{n}$  queries with  $\tilde{O}(\sqrt{n})$  error

# Differential Privacy: A Tutorial

- Basic composition  
Answering **small** numbers of queries
- Advanced composition  
Answering **moderate** numbers of queries
- Coordinated mechanisms  
Answering **huge** number of queries
- Example of Mixing MPC and DP for passwords

# Advanced Composition [DRV10]

Composing  $k$  algorithms, each  $\varepsilon_0$ -DP:

$$\varepsilon_g = O\left(\sqrt{k \cdot \ln \frac{1}{\delta_g}} \cdot \varepsilon_0 + k \cdot \varepsilon_0^2\right)$$

with all but  $\delta_g$  probability.

Simultaneously

Compare with:  $\varepsilon_g = k \cdot \varepsilon_0$  (basic composition)

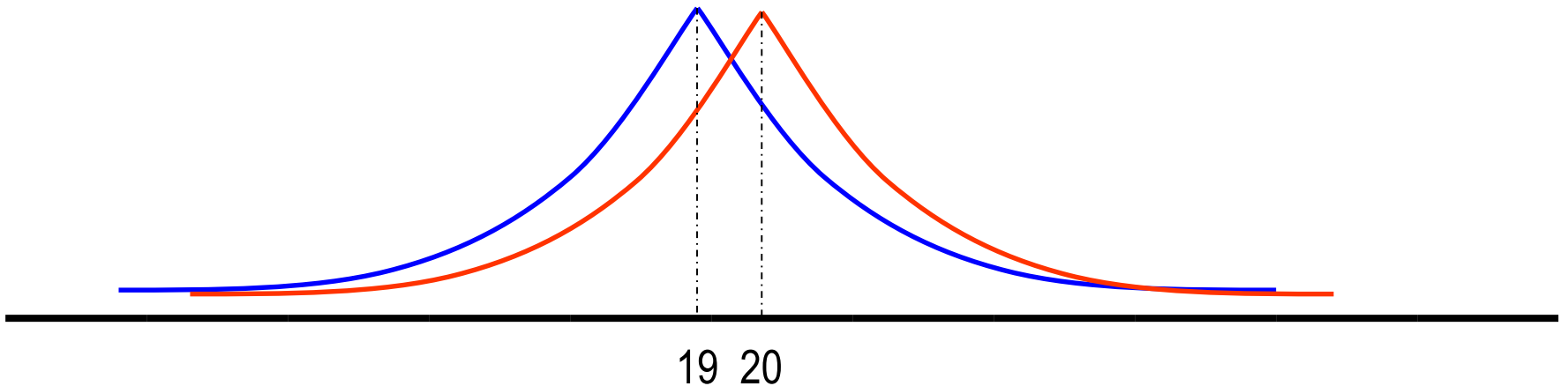
(think of  $k < 1/\varepsilon_0^2$ )

# Privacy Loss

Fix adjacent  $D, D'$ , draw  $y \leftarrow M(D)$

$$\text{PrivacyLoss}(y) = \ln \left[ \frac{\Pr[M(D) = y]}{\Pr[M(D') = y]} \right]$$

Can be positive, negative (or infinite)



# Privacy Loss

Fix adjacent  $D, D'$ , draw  $y \leftarrow M(D)$

$$\text{PrivacyLoss}(y) = \ln \left[ \frac{\Pr[M(D) = y]}{\Pr[M(D') = y]} \right]$$

- random variable, has a mean
- $(\varepsilon, 0)$  –  $DP$ : w.p. 1 over  $y$ ,  
 $|\text{PrivacyLoss}(C)| \leq \varepsilon$
- $(\varepsilon, \delta)$  –  $DP^*$ : w.p.  $1 - \delta$  over  $y$ ,  
 $|\text{PrivacyLoss}(C)| \leq \varepsilon$

# Advanced Composition [DRV10]

Composing  $k$  algorithms, each  $\epsilon_0$ -DP:

$\epsilon_g = \epsilon_0 \sqrt{k}$   
Fundamental law of information recovery [DN03]:

with all but  $\delta_g$  accuracy: **Must have error  $\Omega(\sqrt{n})$**

- Better comp **For all  $\delta_g$  simultaneously**
- Answer  $n$  queries, error  $\tilde{O}(\sqrt{n \cdot \ln(1/\delta_g)})$ 
  - independent Laplace noise
- Will see: Answer  $k$  queries, error  $\tilde{O}(\sqrt{\log k \cdot n \cdot \ln(1/\delta_g)})$ 
  - **coordinated noise** - Private Multiplicative Weights [HR10]



# Advanced Composition Proof

If  $M$  is DP, then privacy loss RV has:

- $E[\text{PrivacyLoss}(C)] = O(\varepsilon^2)$  (down to  $\varepsilon^2/2$  [DR15])
- $|\text{PrivacyLoss}(C)| \leq \varepsilon$

Model **cumulative loss** from  $M_1 \dots M_k$  as **Martingale**

$$\Pr \left[ \left( \sum_{i=1}^k \text{Loss}(C_i) \right) > k\varepsilon^2 + \sqrt{k}\varepsilon \cdot t \right] \leq \exp(-t^2/2)$$



# Advanced Composition of $(\epsilon, \delta)$

Composing  $k$  algorithms, each  $(\epsilon_0, \delta_0)$ -DP:

$$\epsilon_g = O\left(\sqrt{k \cdot \ln \frac{1}{\delta_{err}} \cdot \epsilon_0 + k \cdot \epsilon_0^2}\right)$$

with all but  $\delta_g = \delta_{err} + k \cdot \delta_0$  probability.

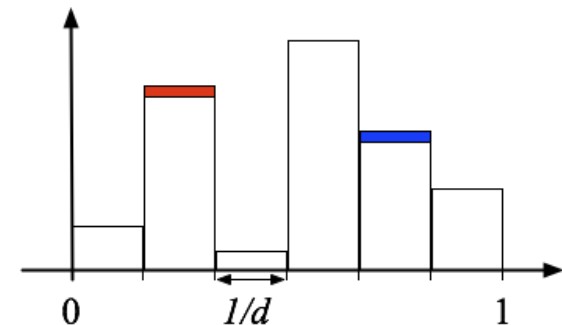
Generally:  $\delta$ 's add up

# Do Better for Some Query Sets?

Use sensitivity of answer **vector** [DMNS06]

## Example:

- Histograms, divide  $U$  into  $d$  **disjoint** bins  $S_1 \dots S_d$   
 $d$  queries:  $q_i$  counts #users in set  $S_i$
- For adjacent  $D, D'$ , only two answers can change, and each can change by **1**
- Global sensitivity of answer vector is **2**
- Add only  $Lap(2/\epsilon)$  noise to each query, still get  $\epsilon$ -privacy



# Further Work: Not Today

Some queries have **high global sensitivity**,  
but (usually) **low local sensitivity**

– Example: Median

Want noise  $\sim$  **local sensitivity**

**Problem:** local sensitivity is itself sensitive!

**Smooth sensitivity** [Nissim Raskhodnikova Smith 08]

Compute “smoothed” diffP upper-bound on **local sensitivity**

**Propose Test Release** [Dwork-Lei 09]

diffP test of local sensitivity, fail if too high

# Differential Privacy: A Tutorial

- Basic composition  
Answering **small** numbers of queries
- Advanced composition  
Answering **moderate** numbers of queries
- Coordinated mechanisms  
Answering **huge** number of queries

# $k$ -fold **composition** of $\varepsilon_0$ -differential privacy?

Answer 1  
[DMNS06]

$\varepsilon_0 k$ -differential privacy

Answer 2  
[DRV10]

$\varepsilon_0 \sqrt{k}$ -differential privacy

Note: for small enough  $\varepsilon_0$

can (privately) answer  $n$  queries with error  $\sqrt{n}$

# Do better for general queries?

## Negative Results

Cannot answer  $n$  counting queries with error  $o(\sqrt{n})$   
[DiNi03, DwMcTa07, DwYe08]

In all these cases: **strong privacy violation**

What **can** we do?



almost entire DB  
compromised

# Many Queries Question

- Can we achieve error  $\sqrt{n}$  for  $k \gg n$  queries
- with **any** reasonable notion of privacy

**Yes!**

Can answer **huge** numbers of queries with small error and differential privacy

[BLR08,DNRRV09,DRV10,RR10,HR10]



# The Exponential Mechanism

Sometimes adding noise makes no sense

e.g. **output is not a number:**

- minimum cut in a graph
- decision tree classifier

[McSherry-Talwar 2007] Motivation: auction design

- DP implies approximate **truthfulness**

Subsequently applied broadly and successfully

- Can phrase any DP Mechanism as an instance of EM

# The Exponential Mechanism

Input  $x$  output  $y$  arbitrary

Define for any possible input  $x$  and output  $y$  some measure of **utility**  $u(y, x)$  - a real number

- The larger  $u(y, x)$  the better the result
- Adjacent databases should have similar scores

$$- \Delta = \max_{x, x', y} |u(x, y) - u(x', y)| \text{ small}$$

The mechanism: on input  $x$ , output  $y$  w.p  $\propto e^{\epsilon u(y, x) / \Delta}$

# Simple Example: Private Lunch Preferences

DB of  $n$  individuals, lunch options  $\{1, 2, \dots, k\}$ ,  
– each individual likes/dislikes each option (1 or 0)

**Goal:** output a lunch option that many like

**Mechanism:**

Output option  $j$  with probability  $\propto e^{\varepsilon \cdot \ell(j)}$

Where  $\ell(j) = \#$  who like  $j$

Actual probability:  $\frac{e^{\varepsilon \cdot \ell(j)}}{\sum_i e^{\varepsilon \cdot \ell(i)}}$



Normalizer

# Private Lunch: $2\varepsilon$ -Privacy

For each option  $j \in [k]$ ,  $\ell(j) = \#$  who like  $j$

**Mechanism:**

Output  $j$  with probability  $\frac{e^{\varepsilon \cdot \ell(j)}}{\sum_i e^{\varepsilon \cdot \ell(i)}}$

For adjacent DBs,  $\forall j: \ell(j)$  can differ by 1

- $e^{\varepsilon \cdot \ell(j)}$  changes by  $\leq e^\varepsilon$  factor
- $(\sum_i e^{\varepsilon \cdot \ell(i)})$  changes by  $\leq e^\varepsilon$  factor

For every  $j$ ,  $\Pr[\text{output} = j]$  changes by  $\leq e^{2\varepsilon}$

# Private Lunch: $\tilde{O}(\log k / \varepsilon)$ – Utility

For each option  $j \in [k]$ ,  $\ell(j) = \#$  who like  $j$

**Mechanism:**

Output  $j$  with probability  $\frac{e^{\varepsilon \cdot \ell(j)}}{\sum_i e^{\varepsilon \cdot \ell(i)}}$

- If  $\ell(j_1) < \ell(j_2) - d$ , prob. of  $j_1$  is  $e^{\varepsilon d}$  **times smaller**
- If  $\ell(j) < \max_i \ell(i) - (\ln k + b)/\varepsilon$ ,  
 $\Pr[\text{output is } j] \leq 1/(k \cdot \exp(b))$
- **Union bound** over all  $j \in [k]$ :  
 $\Pr[\text{answer} < \max - (\ln k + b)/\varepsilon] \leq \exp(-b)$

# Recap

- Notion of  $\epsilon$ -differential privacy and  $(\epsilon, \delta)$ -differential privacy
- **Composition**: basic and advanced
  - For  $k$ -fold **composition** of  $\epsilon_0$ -dp:  $\epsilon_0 k$  and  $\epsilon_0 \sqrt{k}$  respectively
  - Laplace mechanism and answering low sensitivity queries
    - The size of database as bound on number of queries
- The exponential mechanism

# The Exponential Mechanism

Input  $x$  output  $y$  arbitrary

Define for any possible input  $x$  and output  $y$  some measure of **utility**  $u(y, x)$  - a real number

- The larger  $u(y, x)$  the better the result
- **Adjacent** databases should have similar scores

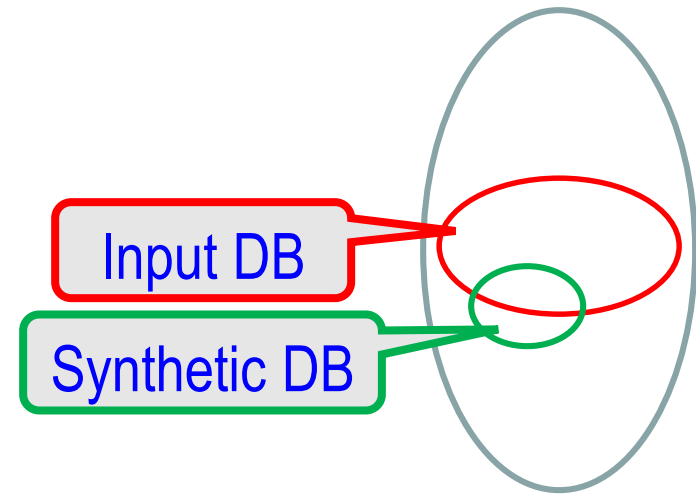
$$- \Delta = \max_{x, x', y} |u(x, y) - u(x', y)| \text{ small}$$

The mechanism: on input  $x$ , output  $y$  w.p  $\propto e^{\epsilon u(y, x) / \Delta}$

# Answering Many Queries [BLiRo08]

Answer **any set**  $Q$  of counting queries with diffP

- Error is  $\tilde{O}(n^{2/3} \log^{1/3} |Q|)$ 
  - uses exponential mechanism
- algorithm outputs **synthetic DB**
  - Output is a (small) DB itself



Hope for rich private analysis of small DBs!

$\#queries \gg DB\ size$

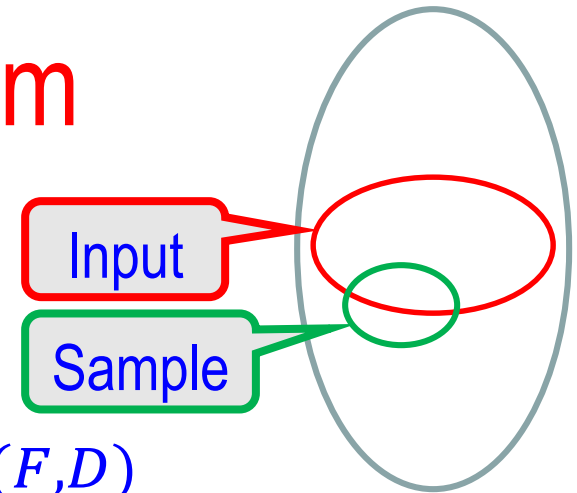


# The BLR Algorithm

Input DB  $D$  of size  $n$ , query set  $Q$ :

**Sample** DB of size  $m$ : ( $m < n$ )

DB  $F$  gets picked w.p.  $e^{-\varepsilon \cdot \text{dist}(F,D)}$



For DBs  $F$  and  $D$

$$\text{dist}(F, D) = \max_{q \in Q} |q(F) - q(D)|$$

Utility

**Intuition:** far DBs get smaller probability

Two samples that are **approximately the same** on  $Q$  get the **same weight**

# The BLR Algorithm: $2\varepsilon$ -Privacy

Input DB  $D$  of size  $n$ , query set  $Q$ :

**Sample** DB of size  $m$ : ( $m < n$ )

DB  $F$  gets picked w.p.  $e^{-\varepsilon \cdot \text{dist}(F,D)}$

---

For adjacent  $D, D'$  for every  $F$

$$|\text{dist}(F, D) - \text{dist}(F, D')| \leq 1$$

- Probability of  $F$  by  $D$ :

$$e^{-\varepsilon \cdot \text{dist}(F,D)} / \sum_{G \text{ of size } m} e^{-\varepsilon \cdot \text{dist}(G,D)}$$

- **Probability of  $F$  by  $D'$ :**

– **numerator** and **denominator** can change by  $e^\varepsilon$

$2\varepsilon$ -differential privacy

# BLR Algorithm: Error

Input DB  $D$  of size  $n$ , query set  $Q$ :

**Sample** DB of size  $m$ : ( $m < n$ )

DB  $F$  gets picked w.p.  $e^{-\varepsilon \cdot \text{dist}(F,D)}$

Fix desired error  $\alpha$ ,  $m = \tilde{O}((n/\alpha)^2 \cdot \log|Q|)$

- $\exists F^*$  of size  $m$  with  $\text{dist}(F^*, D) \leq \alpha$   
 $\Pr[F^*] \propto e^{-\varepsilon\alpha}$

A random sample is good whp!

- $\forall F$  bad with  $\text{dist} 2\alpha$ ,  $\Pr[F_{bad}] \propto e^{-2\varepsilon\alpha}$
- $\sum_{F_{bad}} \Pr[F_{bad}] \propto |U|^m \cdot e^{-2\varepsilon\alpha}$

Take  $\alpha = \tilde{O}(n^{2/3} \cdot \log^{1/3} |Q|)$ ,

$$\Pr[F_{good}] \gg \sum \Pr[F_{bad}]$$

# BLR Algorithm: Running Time

Input DB  $D$  of size  $n$ , query set  $Q$ :

**Sample** DB of size  $m$ : ( $m < n$ )

DB  $F$  gets picked w.p.  $e^{-\varepsilon \cdot \text{dist}(F,D)}$

---

**Brute-force sampling:**

Need to enumerate **every** size- $m$  database,

where  $m = \tilde{O}((n \setminus \alpha)^2 \cdot \log |Q|)$

Running time  $\approx |U|^{\tilde{O}((n \setminus \alpha)^2 \cdot \log |Q|)}$

# BLR algorithm: Conclusion

## Offline algorithm

- Error:  $\tilde{O}(n^{2/3} \cdot \log^{1/3} |Q| / \varepsilon)$
- Running time:  $|U|^{\tilde{O}(n^{2/3} \cdot \log^{1/3} |Q| / \varepsilon)}$

## Private Multiplicative Weights Algorithm

### Online algorithm

- Error  $\tilde{O}(\sqrt{n \cdot \log |Q|} / \varepsilon)$
- Running time:  $\text{poly}(|U|, |Q|, n)$

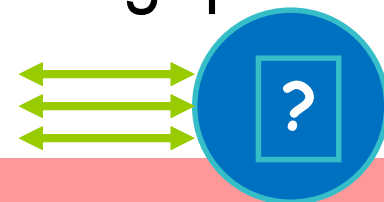
# Key Insight to increasing number of queries: Use **Coordinated Noise**

- If noise is added in with careful coordination,  
rather than independently  
can answer **hugely** many queries

#queries  $\gg$  DB size

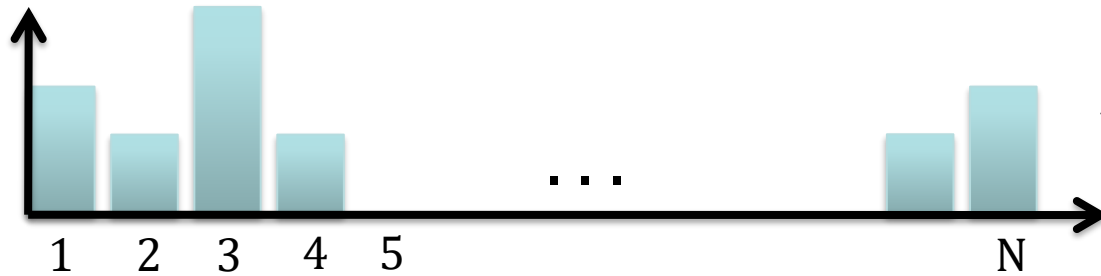
**Wave** of results showing:

- Differential Privacy for **every** set  $\mathcal{Q}$  of counting queries
- Error is  $\tilde{O}(n^{1/2} \log|\mathcal{Q}|)$ 
  - Even in the *interactive* case –
  - Private Multiplicative Weights Algorithm



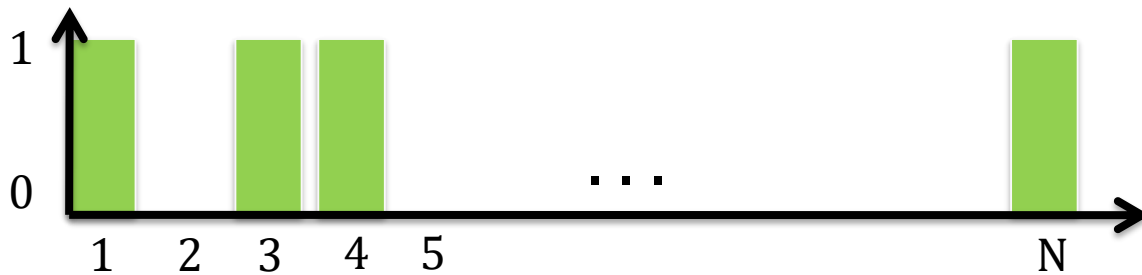
# Represent database as a linear function

Data set  $D$  is *distribution* over universe  $U$ ,  $|U| = N$



$$D[i] = \frac{\text{\#type } i \text{ items in } D}{n}$$

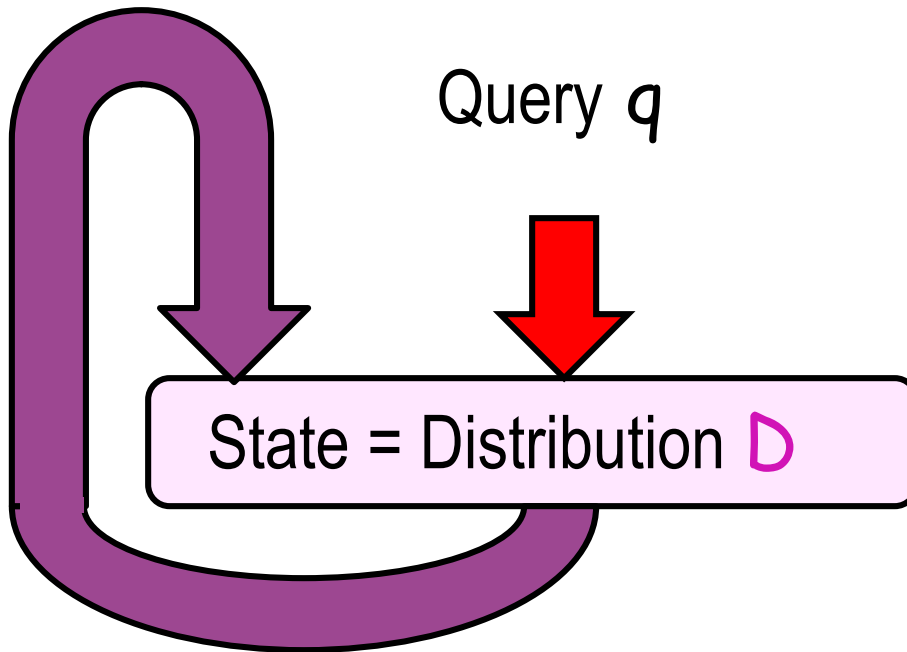
Statistical query  $q$  is vector in  $\{0,1\}^N$



$$q(D) = \langle q, D \rangle \cdot n$$

$$q(D) \in [0,1]$$

# Maintaining State





# The PMW Algorithm

Maintain

Initializ

Repeat



on universe  $U$

This is the state.  
Is completely public!

on  $U$

Algorithm fails if more than  $k$  updates

- Set  $\mu$

• **Repetitive Weights** update occurs:

- Powerful tool in algorithms design

The true value

- Learn a Probability Distribution iteratively

- In each round:  $|\hat{a} - a| \leq \hat{T}$  output  $q(D)$ .

- **Test**: either current distribution is good

the plus or minus are according to the sign of the distribution

- **Else** (update): or get a lot of information of distribution

- Output  $\hat{a}$
- Update distribution
- Update  $D[i] / D[i] e^{\pm T/4q[i]}$  and re-weight.

# Private Multiplicative Weights

**Accuracy:** nearly optimal (in worst case)

**Privacy:** differential privacy

**Runtime:** linear dependence on  $|U|$

- $|U|$  **exponential** in # attributes of data

When can we get  $poly(n)$ ?

Lower bounds based **on tracing traitors**

# Differential Privacy: A Tutorial

- Basic composition  
Answering **small** numbers of queries
- Advanced composition  
Answering **moderate** numbers of queries
- Coordinated mechanisms  
Answering **huge** number of queries
- Example of Mixing MPC and DP for passwords

# Applications/Implementations of Differential Privacy

- US Census Bureau **OnTheMap**: gives researchers access to agency data.
- Google's RAPPOR:
  - Randomized **A**ggregatable **P**rivacy-**P**reserving **O**rdinal **R**esponse
  - Open source
  - Local model
- Apple: big news coverage, commitment to privacy
- Applications to **multiple hypothesis testing**

Enabled collection of data  
**Chrome** avoided before

How to hack Kaggle competitions

**Global vs. local**

# Public policy

- California Public Utilities Commission
  - Smart meters
- Interpreting and implementing FERPA by Differential Privacy

Nissim and Wood

Family Educational  
Rights and Privacy Act

Understand how DP **fits with existing regulatory framework.**

**Problem:** regulatory framework is not mathematically precise,  
Idea of de-identification is hard wired in it.

# Challenges

- Small Datasets
- Massive Composition – global epsilon: event level vs user level
- Work in conjunction with Secure Function Evaluation
- Winning the hearts and minds of policy makers...

# Winning the hearts and minds of policy makers...

- Widen scope of implementation and use.
- Identify what are the next good use cases for DP.
  - Construct DP tools matching best the practices and education of users
  - Explain shortcomings of other methods and benefits of DP
  - Need to figure how DP works as one of the layers in a suit of privacy protections.
  - Less straightforward and intuitive than anonymity/de-identification and its variants