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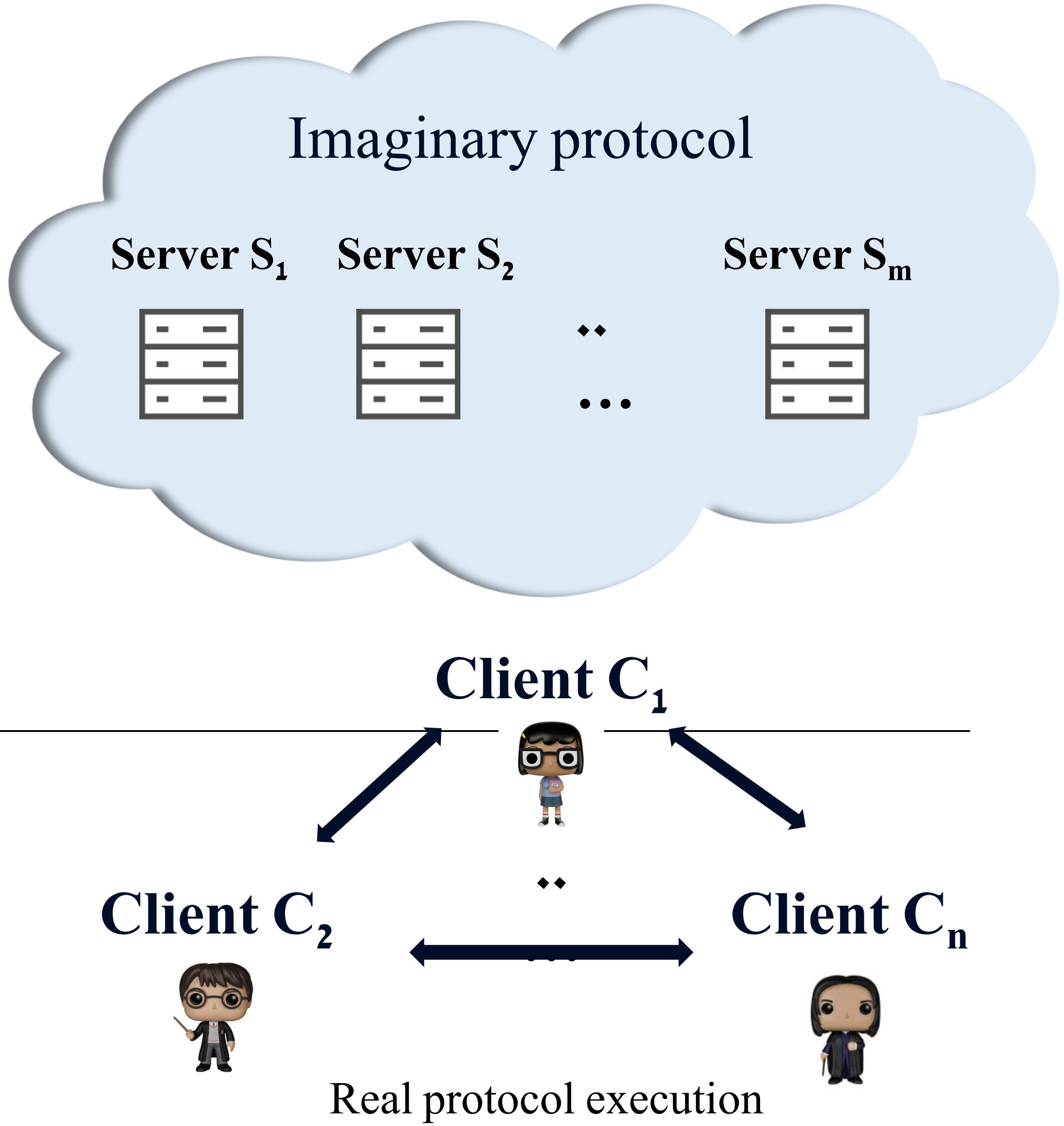
Bar-Ilan University

Practical Instances of MPC-in-the-Head



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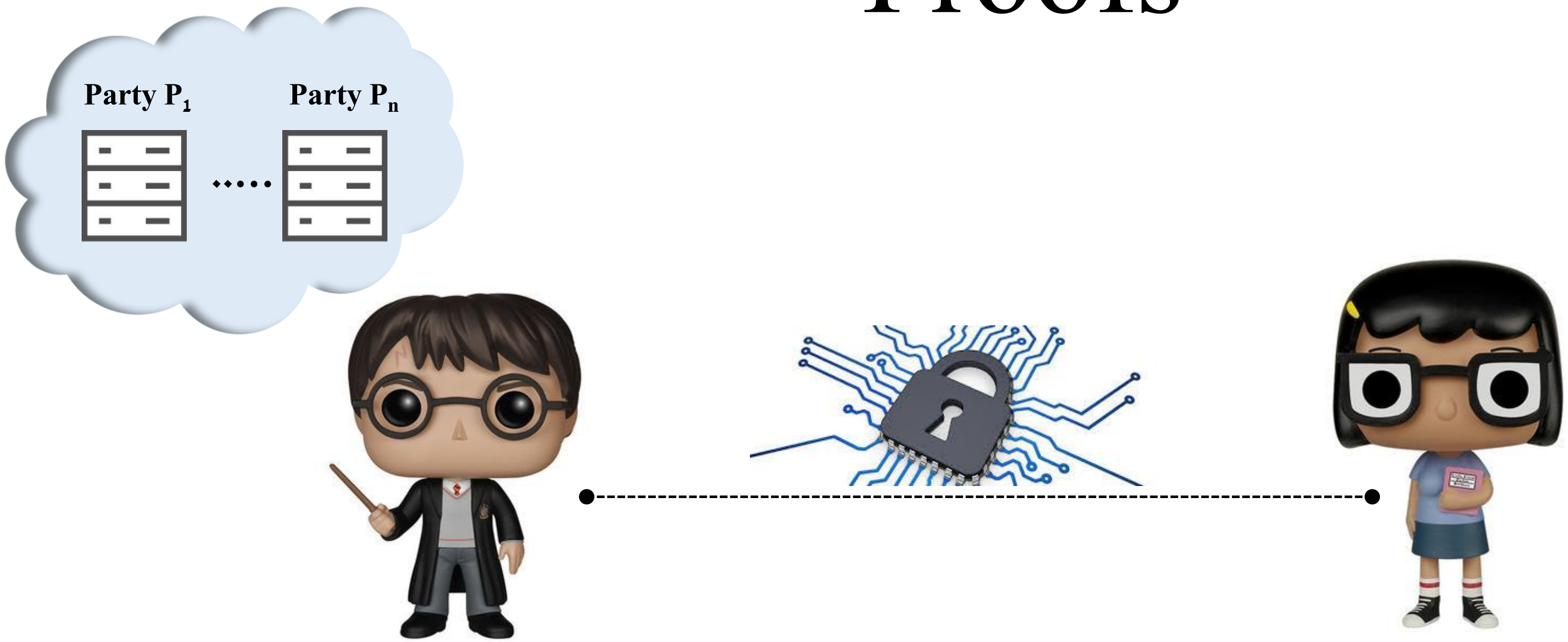
Practical Instances of Zero-Knowledge Proofs



Taxonomy of Proofs

1. P vs NP
2. Interactive vs Non-interactive
3. Trusted setup vs No setup (transparent)
4. ZK vs (only) Soundness
5. Succinct vs Non-succinct
6. Public-Key Crypto vs (only) Symmetric-Key Crypto

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Prior Approaches to "Practical" ZK

1. **Probabilistically Checkable Proofs (PCPs)**
   [BFLS91, Kil92, Mic94, ALMSS98, AS98, DL08, GKR08, GLR11, CMT12, BC12, DFH12, BCCT12, IMS12, Tha13, VSBW13], Interactive PCPs [KR08], Interactive Oracle PCPs [BCGT13, BCS16, RRR16, BCGRS16, BBCGGHPRSTV17,BBHR17, ZGKPP17-18,WTSTW18]

2. **Linear PCPs** [IKO07, Gro10, GGPR13, BCIOP13, Gro10, Lip12, SMBW12, Lip13, PGHR13, BCGTV13, FLZ13, SBBPW13, Lip14, DFGK14, KPPSST14, ZPK14, CFHKKNPZ15, WSRBW15, BCTV14, BBFR15, Groth16, FFGKOP16, BFS16, BISW17, GM17,BBBPWM18]

3. **Multiparty Computation**
   [IKOS07,GMO15,CDGORRSZ17,AHIV17,KKW18]

No setup
High prover's complexity

Short Proofs
Fast Verification
Heavy Public-Key Crypto
Trusted Setup
Quantum Insecure

ZKBoo: Faster Zero-Knowledge for Boolean Circuits [GMO15]

Post-Quantum Zero-Knowledge and Signatures from Symmetric-Key Primitives (ZKB++) [CDGORRSZ17]

Zero-Knowledge Proofs - A Reminder

- Goal: ZK proof for an NP-relation $R(x,w)$

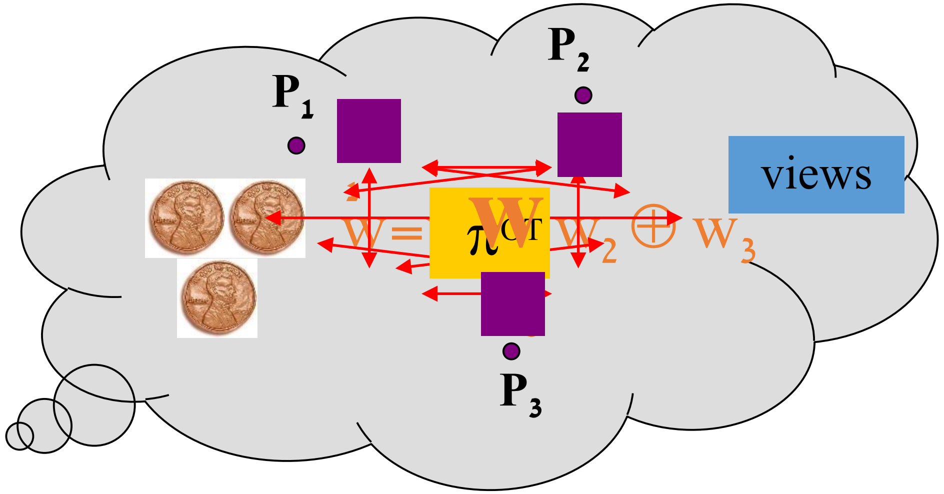
- Towards using MPC:
  - Define n-party functionality

    $g(x;\ w_1,\ldots,w_n) = R(x,\ w_1 \oplus \ldots \oplus w_n)$
- Use OT-based MPC
  - Security in semi-honest model
  - Simple consistency check for dishonest majority when n>2

Zero-Knowledge from 3-Party GMW [IKOS07,GMO15]

Use 3-party GMW protocol π^{OT} for

$g(x;\ w_1,w_2,w_3) = R(x,\ w_1 \oplus w_2 \oplus w_3)$



Prover

Verifier

commit to views V_1,V_2,V_3 →

random i,j ←

open views $V_i,\ V_j$ →

accept iff output=1

&

V_i,V_j are consistent

Soundness error $\leq 2/3$

Prior Approached to “Practical” ZK

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3. **Multiparty Computation**
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No setup
High prover’s complexity

Short Proofs
Fast Verification
Heavy Public-Key Crypto
Trusted Setup
Quantum Insecure

No Setup
Fast Prover
Post Quantum Secure
Everything Linear

Ligero: Lightweight Sublinear Arguments Without a Trusted Setup [AHIV17]

Main Result

Sublinear ZK arguments without trusted setup

- Simple, concretely efficient
- Symmetric-crypto only (eg, SHA256)
- Post-Quantum Secure

First "sublinear" arguments for NP that avoid both complex PCP machinery and public-key crypto

Main Result

Sublinear ZK arguments without trusted setup

Concretely:

- **40-bit security:** comm. is $0.5\sqrt{|C|}$ kb in the Boolean case
- **80-bit security:** Non-interactive via Fiat-Shamir
- Can handle Boolean or arithmetic circuits
- Prover computation: Merkle Tree ($O(\sqrt{|C|})$ leaves) +

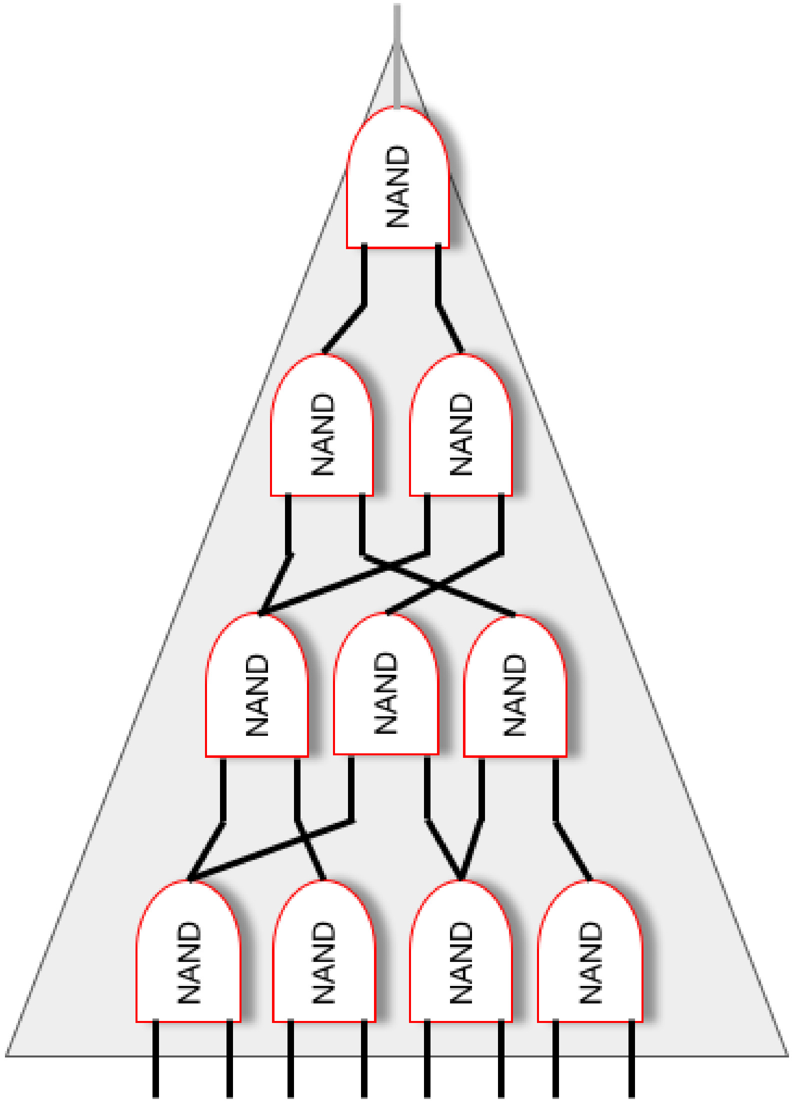
  $O\left(\sqrt{|C|}\right)$ FFT's of $O\left(\sqrt{|C|}\right)$ evaluations

Eg, SHA256 certification with 40-bit security:

i.e. For statement y, prover proves knowledge of x such that SHA256(x) = y

	Linear PCP [Pinocchio]	ZKBoo/++ [CDGORRSZ17]	**Ligero**
Communication	~ bytes	200 KB	**34 KB**
Prover time	mins	~33ms	**140ms**
Verif. time	<10ms	~38ms	**60ms**
Asymptotic Communication	~ bytes	O(|C|)	**$O(\sqrt{\|C\|})$**
Trusted Setup	YES	NO	**NO**
Amortization	NA	NO	**YES**

Proof Schematic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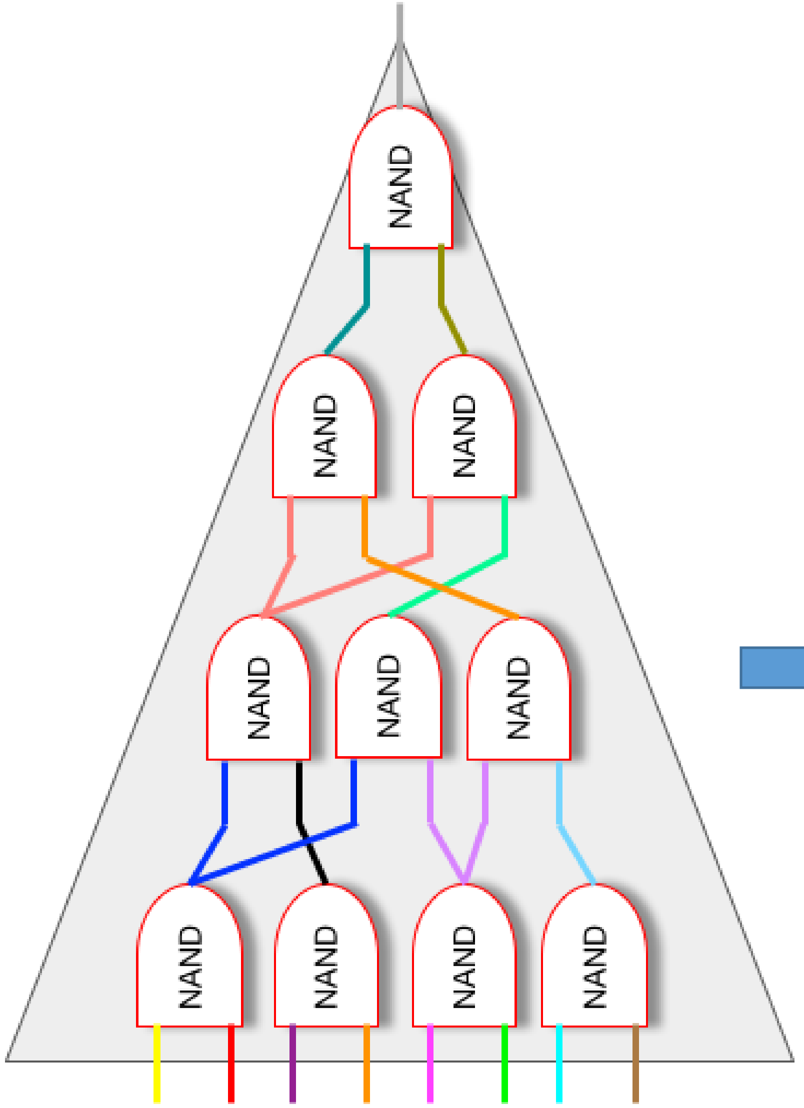
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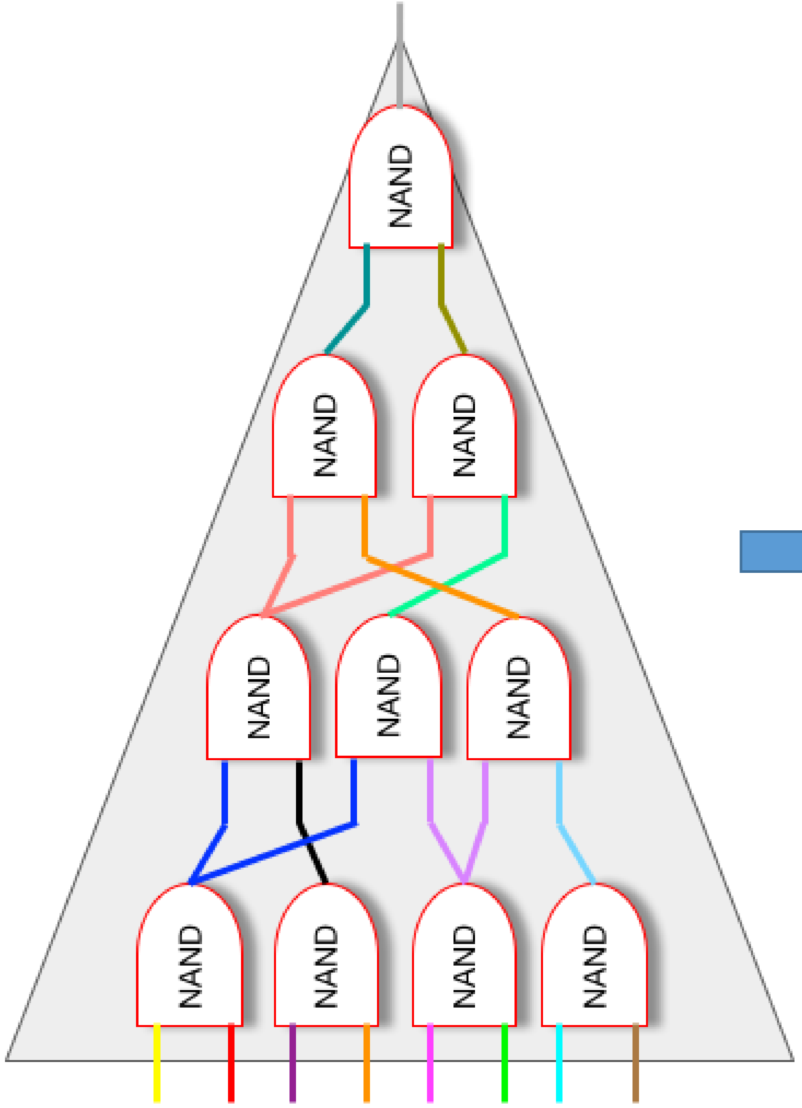
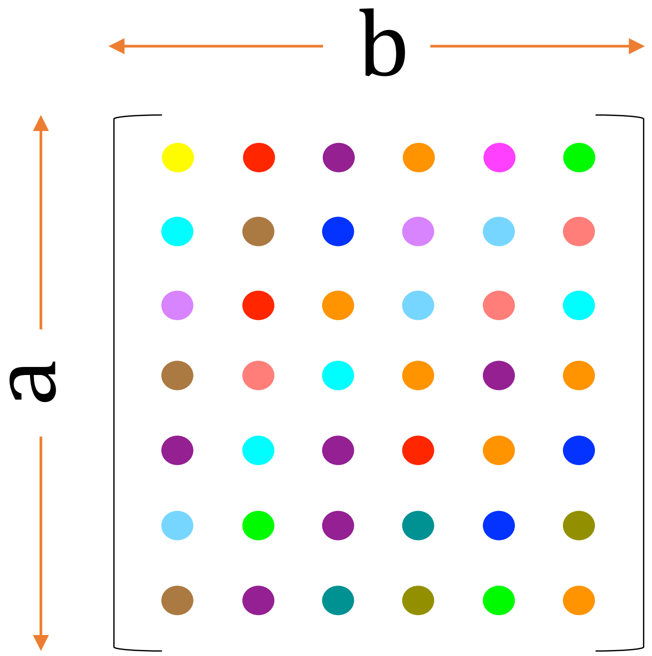
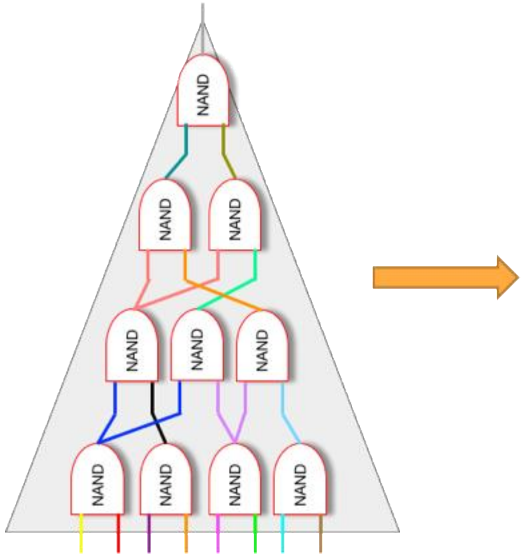
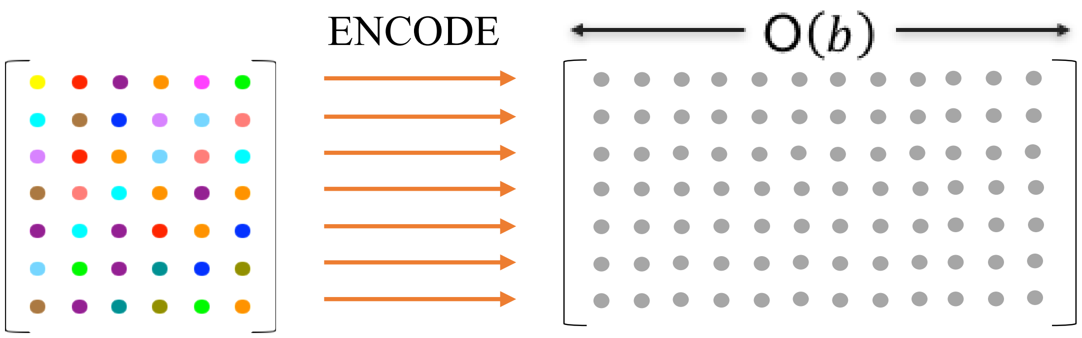




$$a \cdot b \geq X \cdot \#gates$$

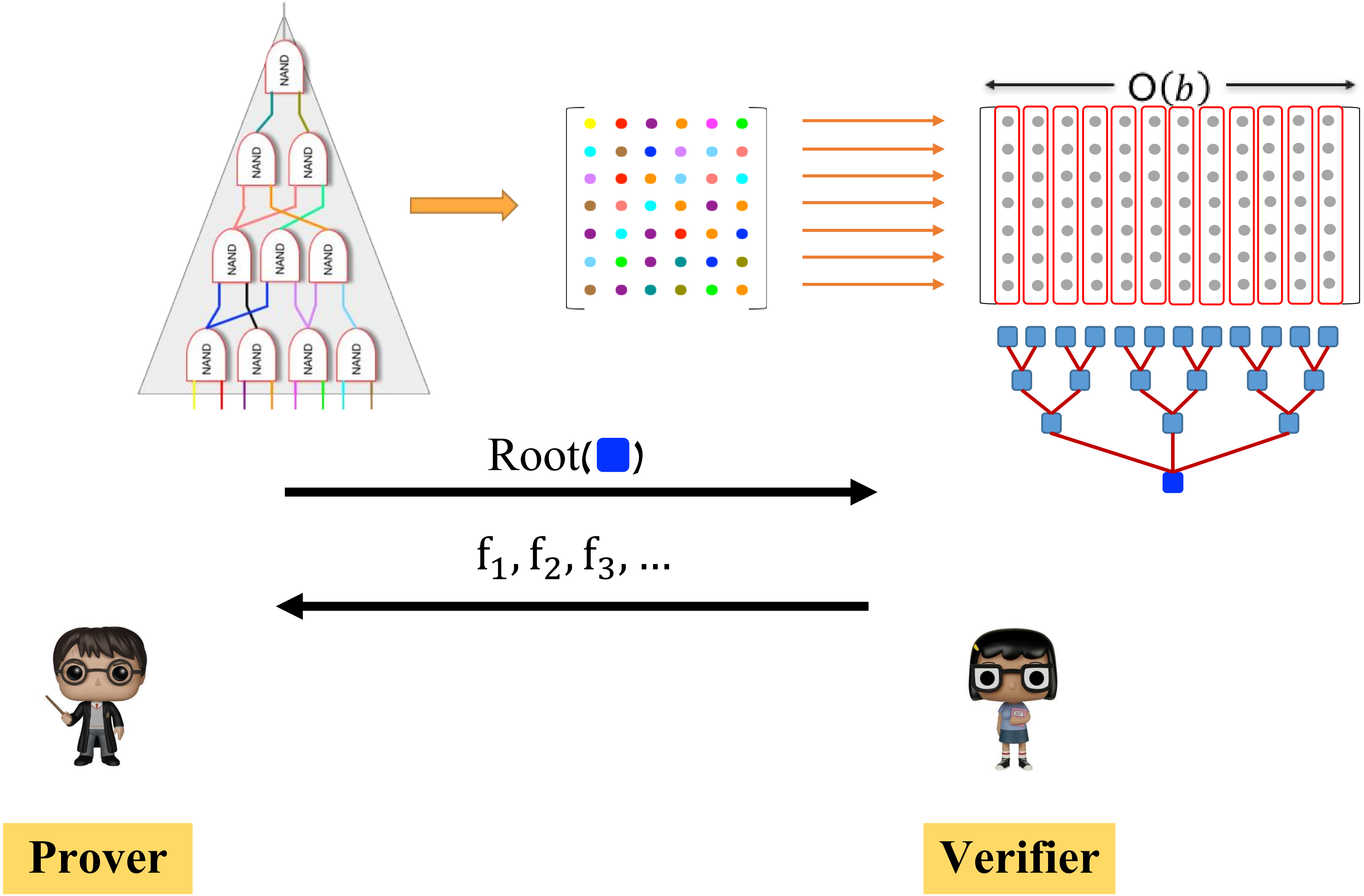
Boolean: X = 2, AND/XOR
Arithmetic: X = 3, AND

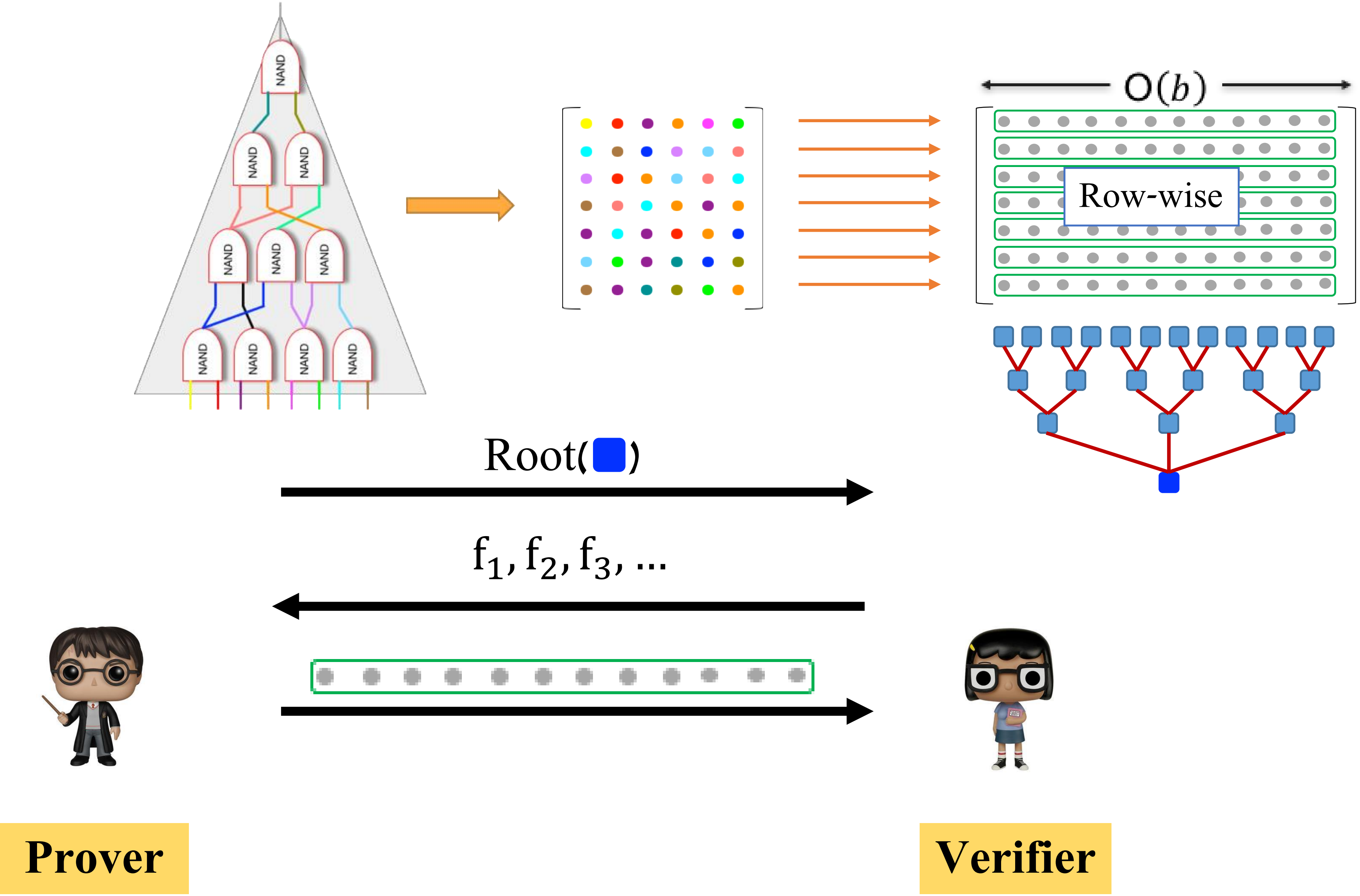


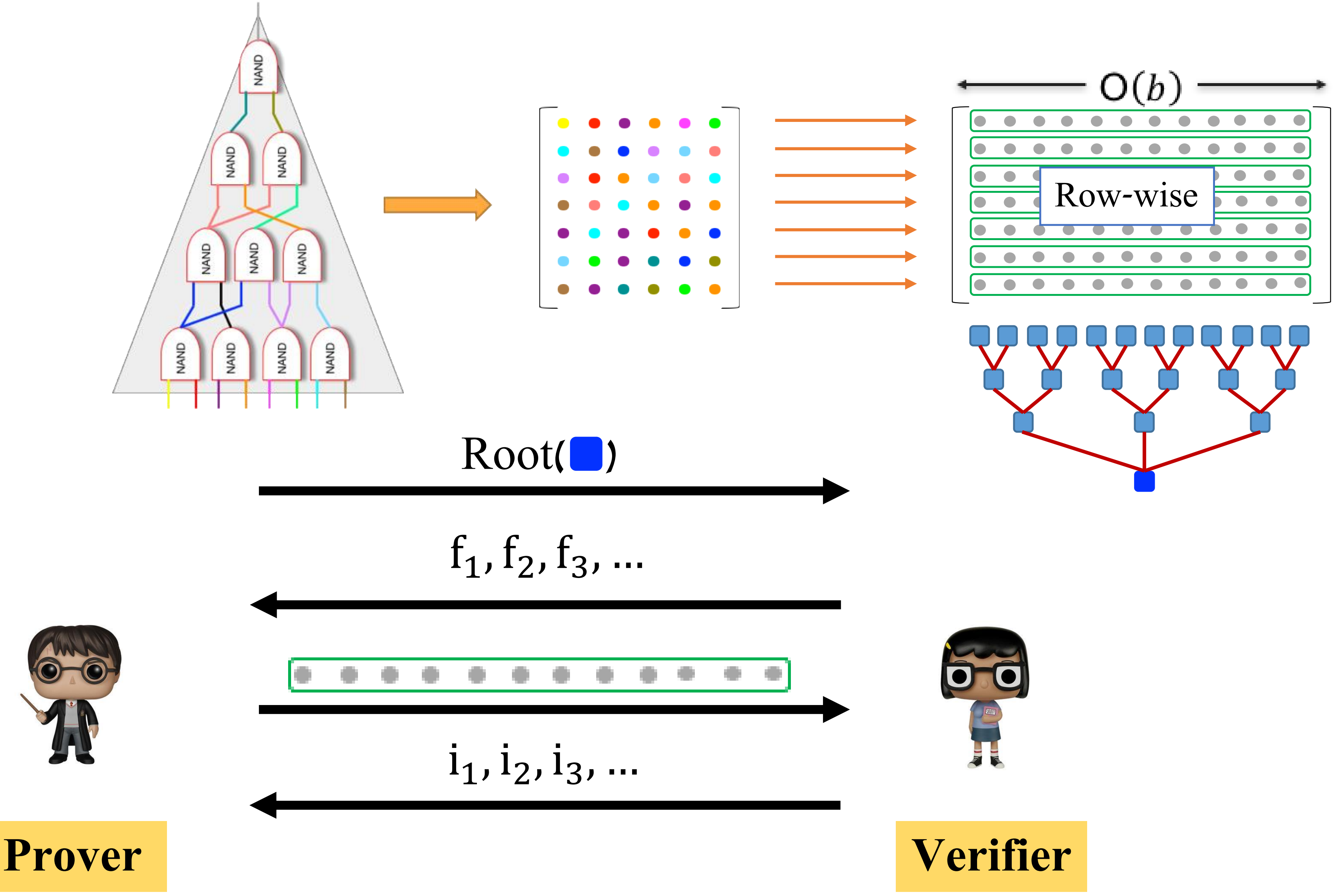


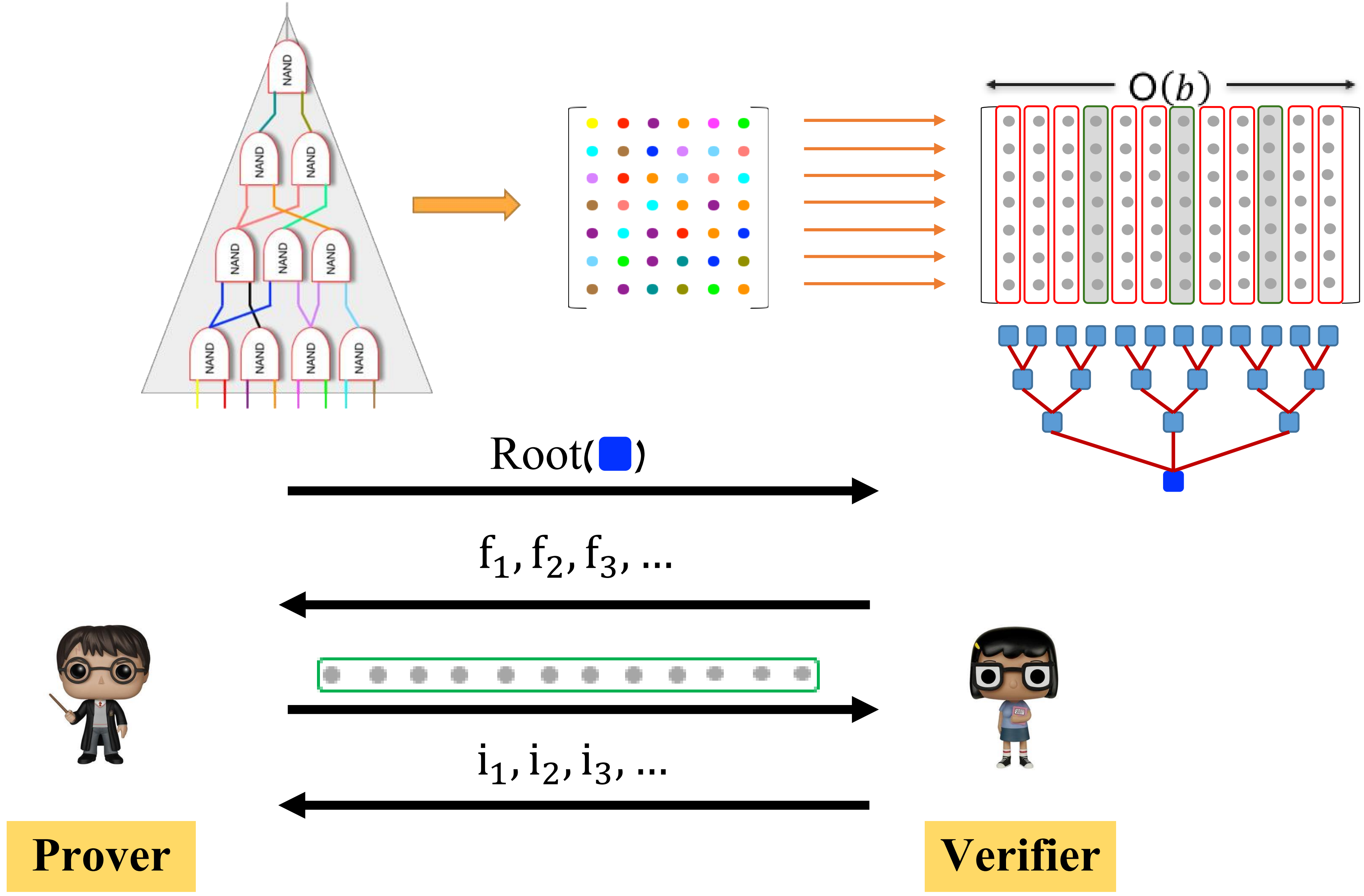
Prover

Verifier


NAND
O(b)
Root(■)
f1, f2, f3, ...
Prover
Verifier


NAND
O(b)
Row-wise
Root( )
f1, f2, f3, ...
Prover
Verifier


NAND
O(b)
Row-wise
Root( )
f_1, f_2, f_3, ...
i_1, i_2, i_3, ...
Prover
Verifier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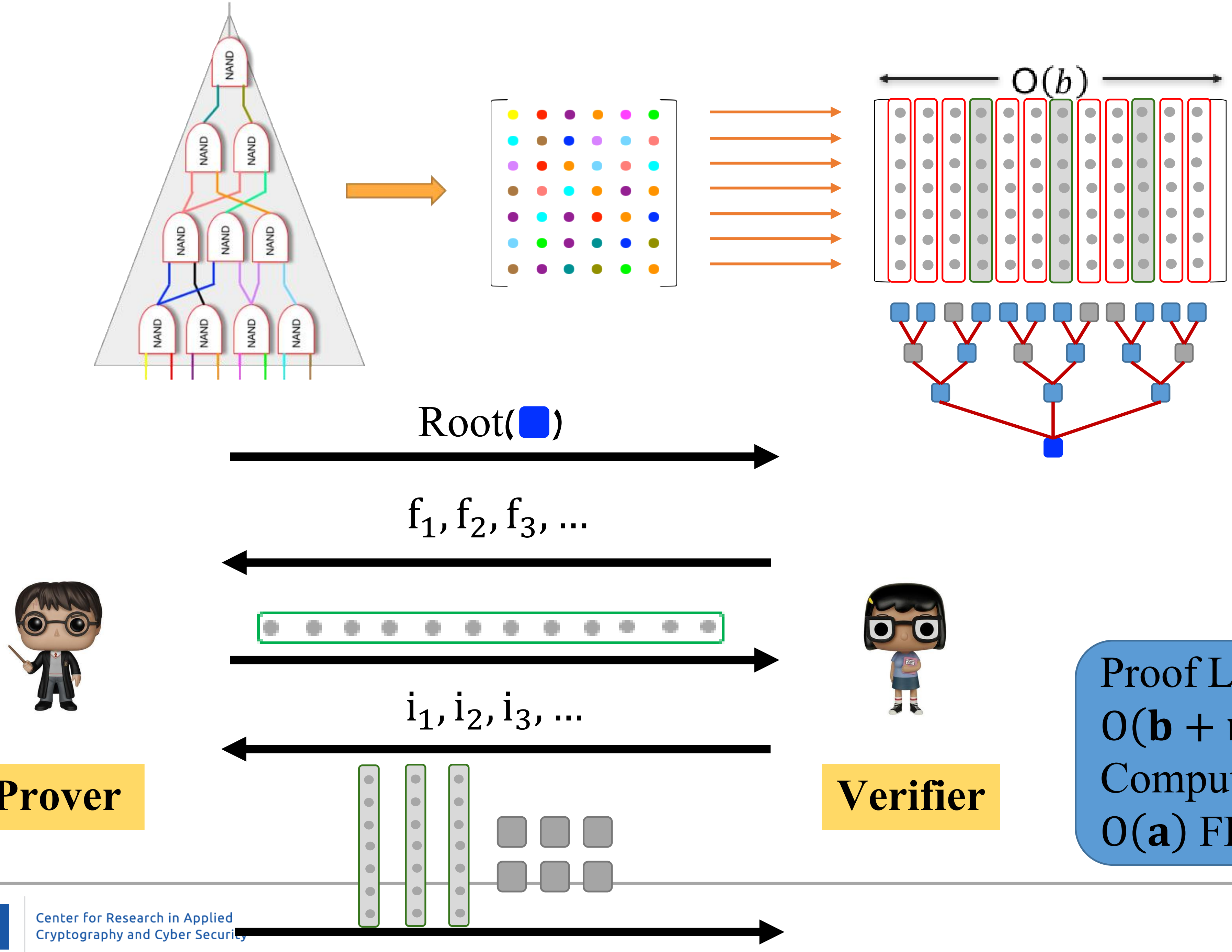

Proof Length:
$O(\mathbf{b} + \boldsymbol{\kappa} \cdot \mathbf{a})$
Computation:
$O(\mathbf{a})$ FFTs of $O(\mathbf{b})$

High-Level Overview

High level approach: use **MPC in the head** [IKOS07]

- Transform Honest-majority MPC to ZK
- Optimized and implemented in [GMO16,CDGORRSZ17]

Can the communication be sublinear?
Communication complexity of (i.t.) MPC > circuit size

Key insight: Communication per party can be sublinear [DI06,IPS09]

High-Level Overview

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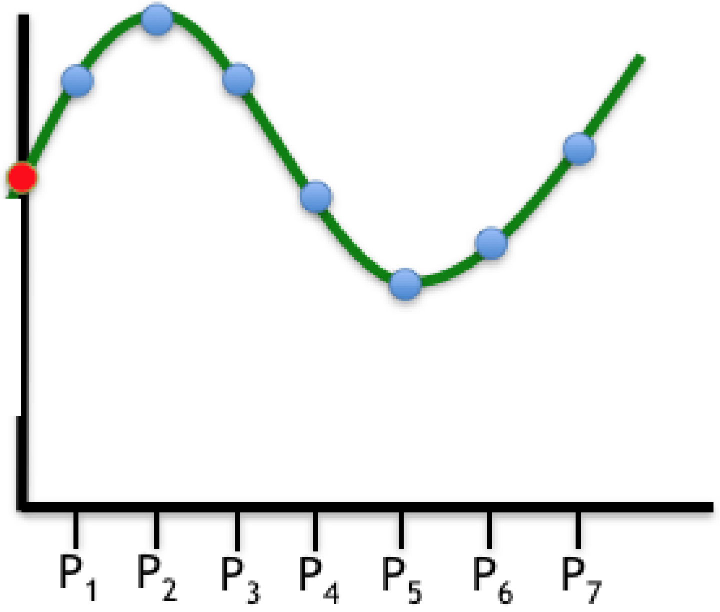
MPC ⟶ Interactive PCP[KR08] $\xrightarrow{\text{[BCS16]}}$ ZK

it size

Key insight: Communication per party can be sublinear [DI06,IPS09]

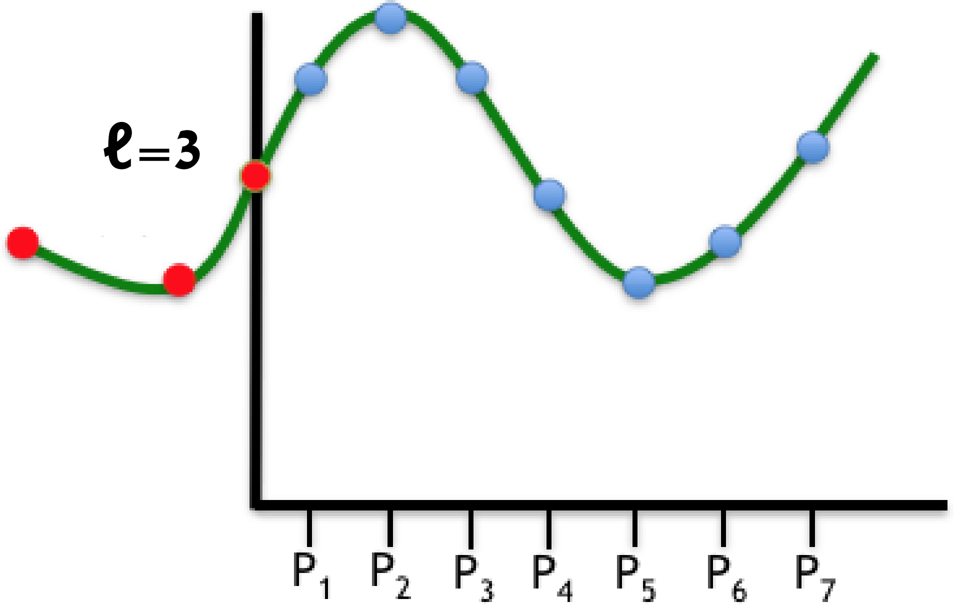
Idea 1: Shamir Secret Sharing [S79]

Pick a random **t**-degree polynomial p such that
p(0) is secret
Distribute p(1), …, p(n)
t shares do not reveal the secrets



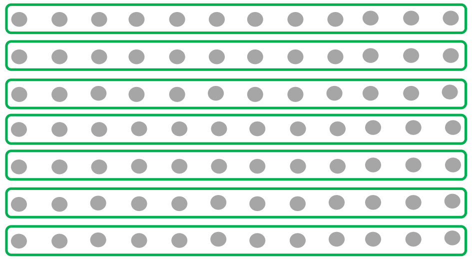
Idea 1: Packed Secret Sharing [FY92]

Pick a random **t+ℓ**-degree polynomial p such that
p(0), p(-1), …, p(-ℓ) are secrets
Distribute p(1), …, p(n)
t+ℓ shares do not reveal the secrets



Idea 2: IPCP for Testing Interleaved RS Codes

Prover



Verifier

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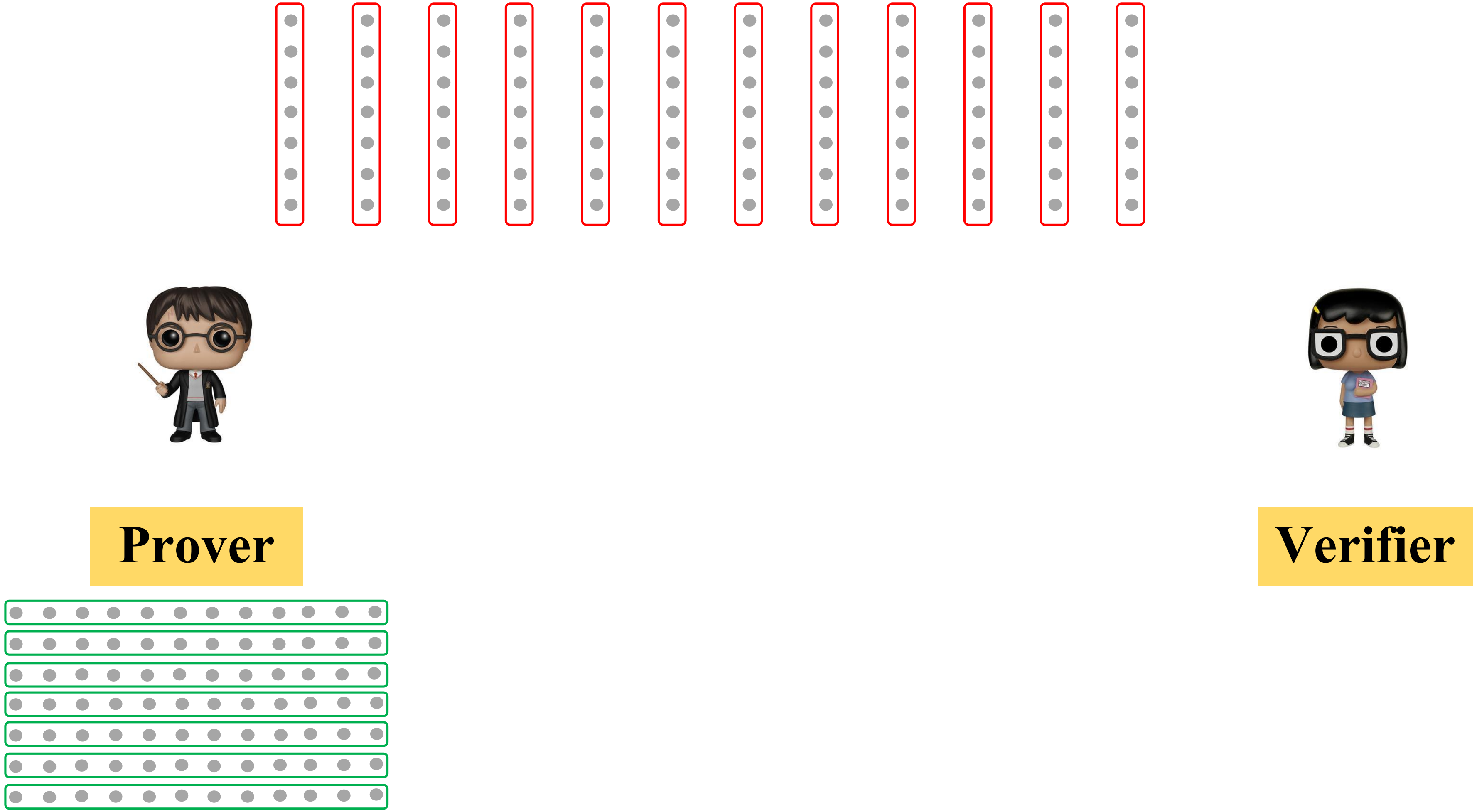

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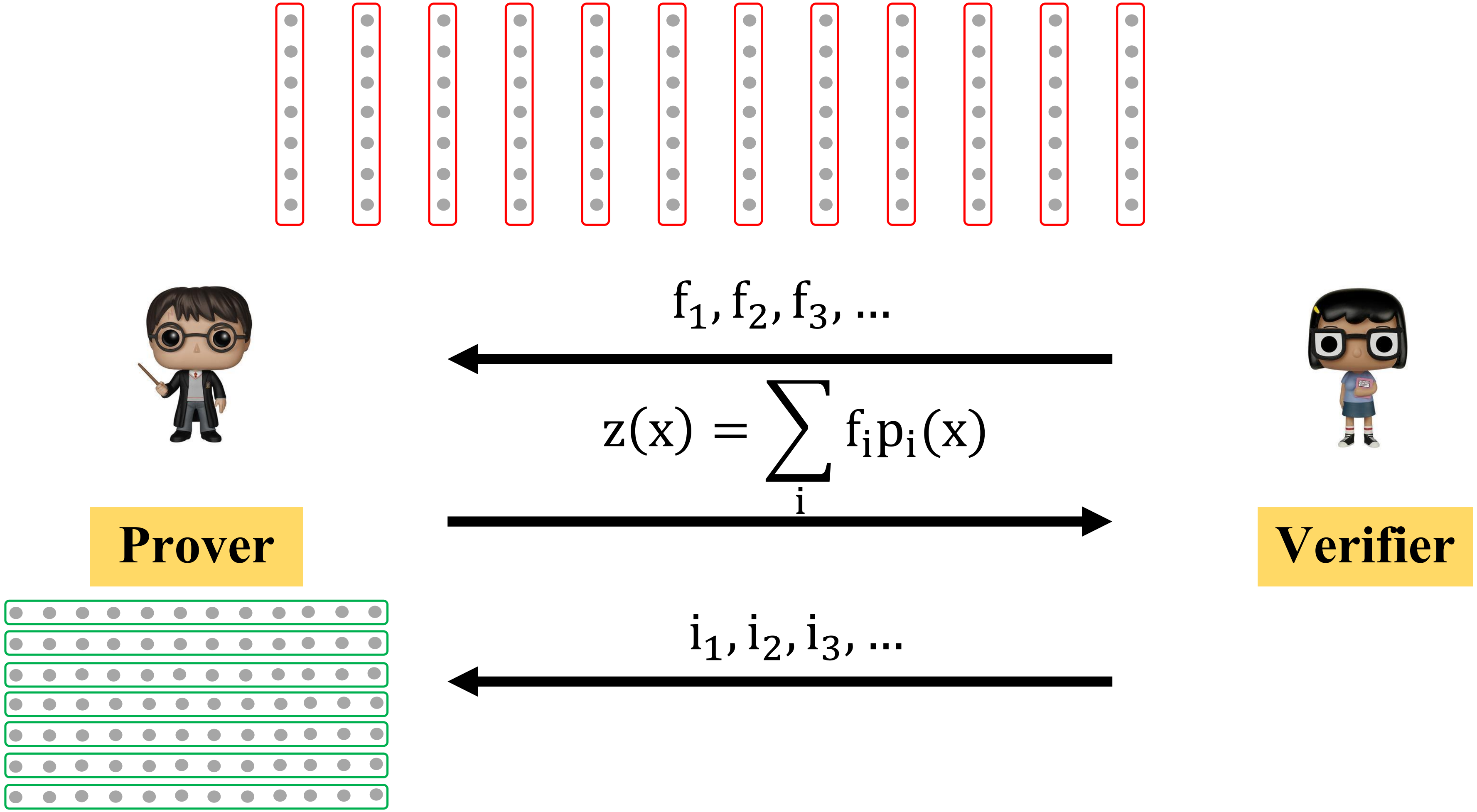

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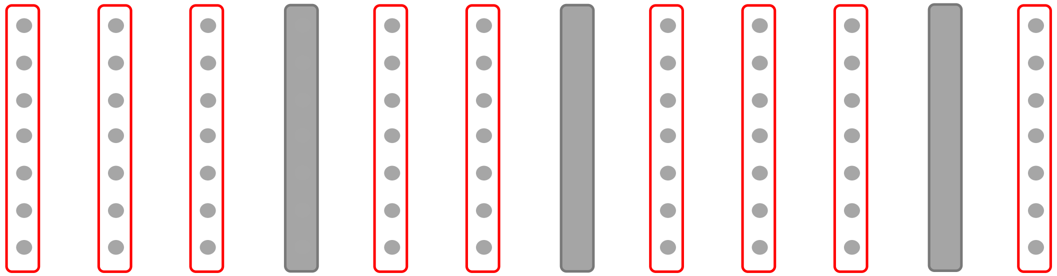
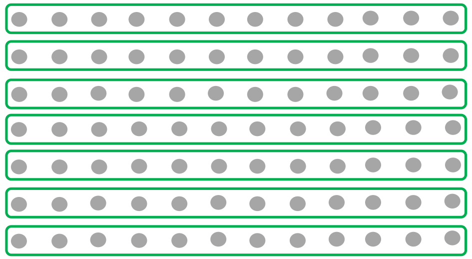


$f_1, f_2, f_3, \ldots$

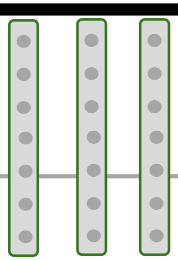
$$z(x) = \sum_i f_i p_i(x)$$

Prover

Verifier



$i_1, i_2, i_3, \ldots$



Check

- $z(x)$ is of degree $t+\ell$
- $z(i) = \sum_i f_i p_i(i)$

Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures [KKW18]

High-Level Overview [KKW18]

Use MPC-in-the-head in the **preprocessing model**

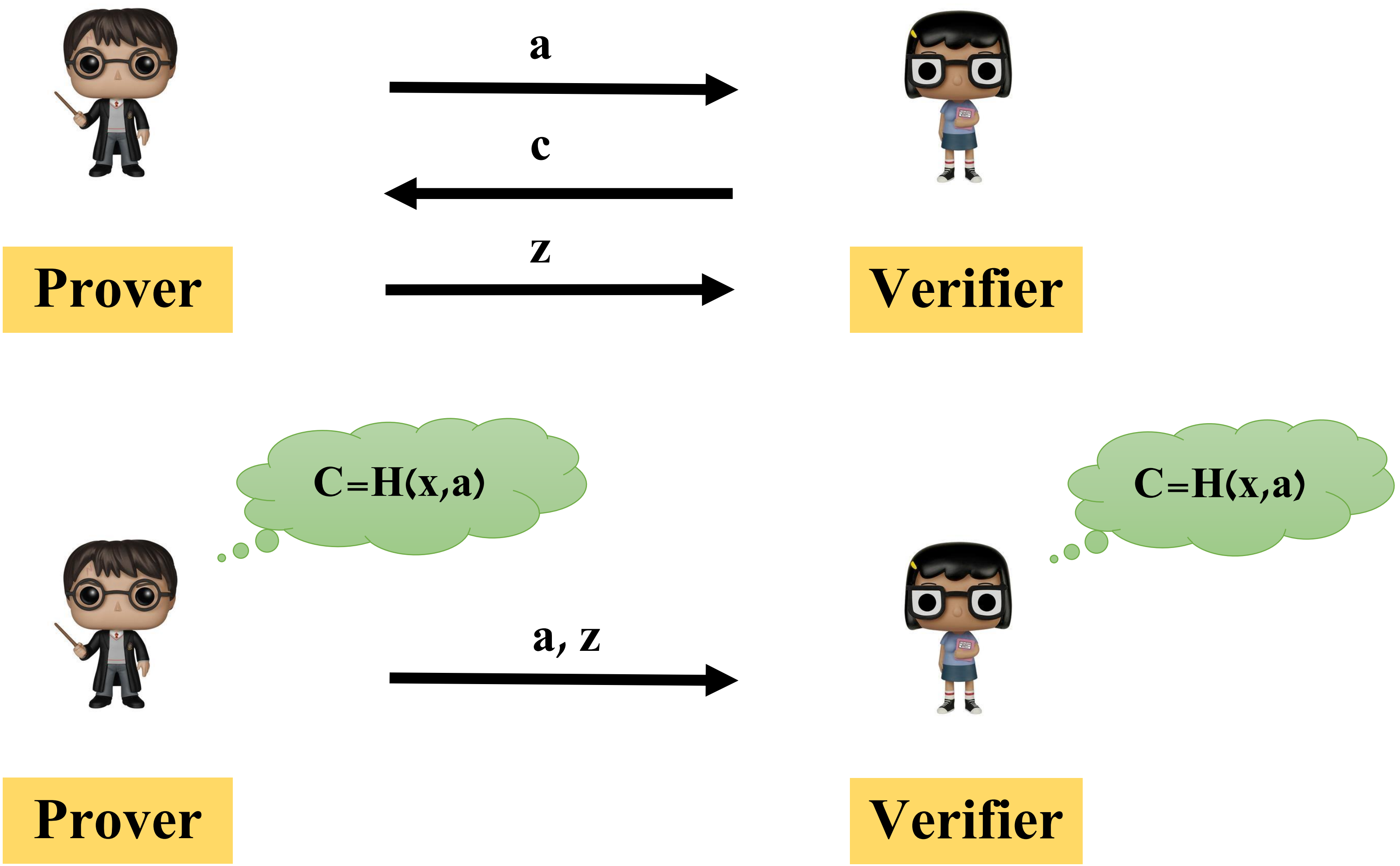
- Check consistency of preprocessing using cut-and-choose

MPC-in-the-head can be instantiated with dishonest majority protocols

- Semi-honest instances for generating correlated randomness
- Implies two versions of 5/3 rounds

Improves on Ligero proof size for circuits containing ≈300-100,000 AND gates

Removing Interaction via the Fiat-Shamir Transform



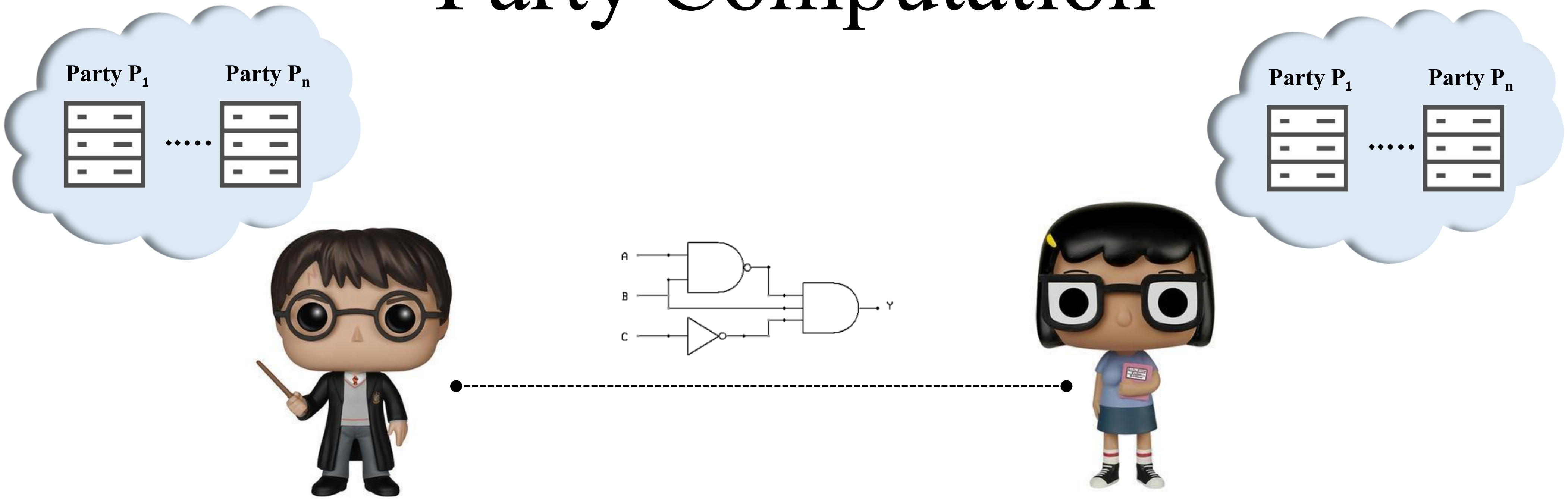
Analysis can be extended to any constant round public-coin protocol [BCS16]

Obtaining (Post Quantum) Signatures from NIZKPoK

The signature scheme:
PK: $y=PRF_k(0^k)$ where PRF is a block cipher
Sig(m): a proof for (y,k) on a challenge H(a,m)

Based on symmetric-key primitives
Extensions to ring and group signatures

Practical Instances of Arithmetic Two-Party Computation

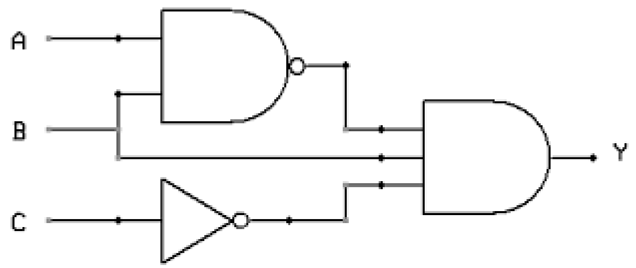


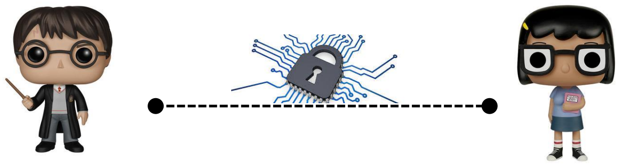
LevioSA: Lightweight Secure Arithmetic Computation from Any Passively Secure OLE [HIMV]

Secure Computation

How is a function represented?

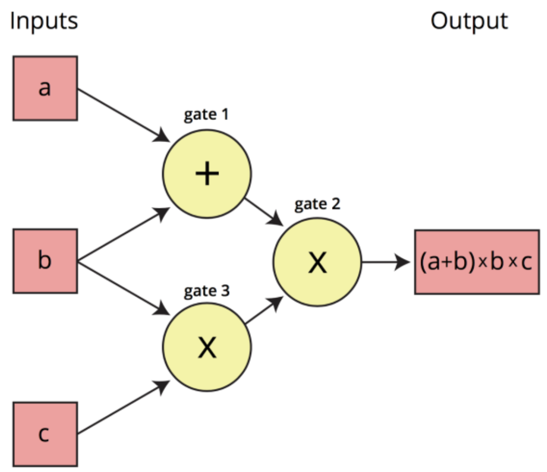
Classically, Boolean circuits [Yao86, GMW87,…]





Arithmetic Computation

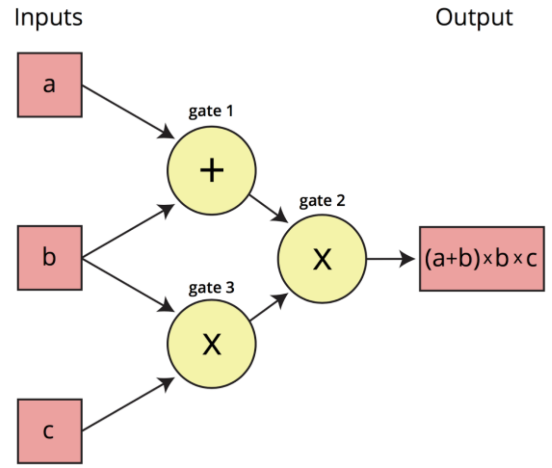
- Many computations are done over an arbitrary field $\mathbb{F}$



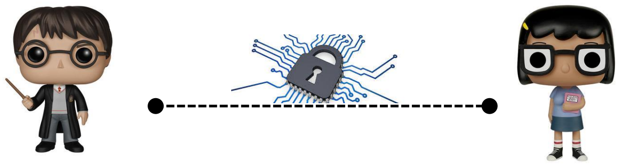
- Notable examples:
  - SHA-256
  - Threshold cryptography [BF97, Gil99…]
  - Machine learning [LP00,…, JVC18, MR18, WCG18]
  - Pattern matching [HL08, HT10, …,KRT17]
  - Even BMR garbling [LPSY15,…]

This Talk

- Two-party
- Active security
- Arithmetic circuits



Goal: reduce communication and computation concrete costs of securely evaluating an arithmetic circuit over a field $\mathbb{F}$ with active security



Atomic Building-Block

- Oblivious linear evaluation (OLE)
  - A generalization of oblivious-transfer

Prior Approaches to Practical Arithmetic 2PC

1. 2PC in the OLE-hybrid [IPS09, DGNNR17]
   - Black-box calls to OLE

22 calls to active OLE

2. 2PC in the OT-hybrid [Gil99, KOS16, FPY18]
   - Black-box calls to OT

6 $\log(|\mathbb{F}|)$ calls to active OT

3. 2PC based on semi-homomorphic encryption [BDOZ11, DPSZ12, KPR18]

Based on concrete (non-standard) assumptions

Main Result

Theorem 1: Actively secure 2PC that makes O(1) invocations of any passive OLE implementation per multiplication

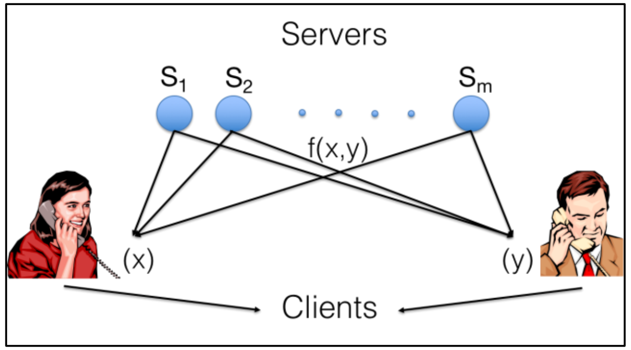
For "nice" circuits our communication overhead is 2

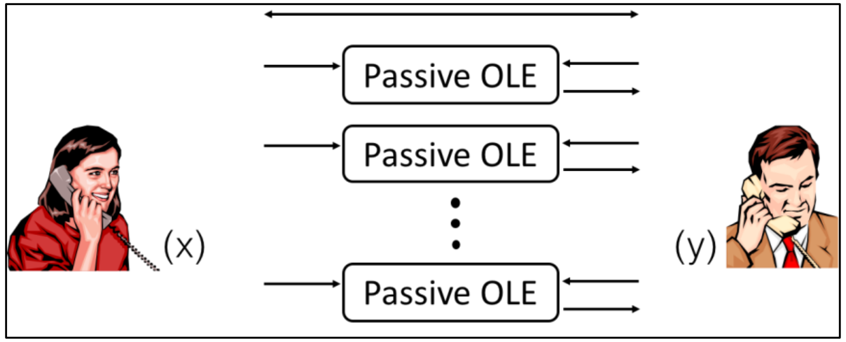
Theorem 2: Actively secure OLE protocol that makes 2 invocations of any passive OLE implementation

[DGNNR17]: 22 black-box calls to any active OLE

[GNN17]: active OLE from 2 calls to specific passive OLE

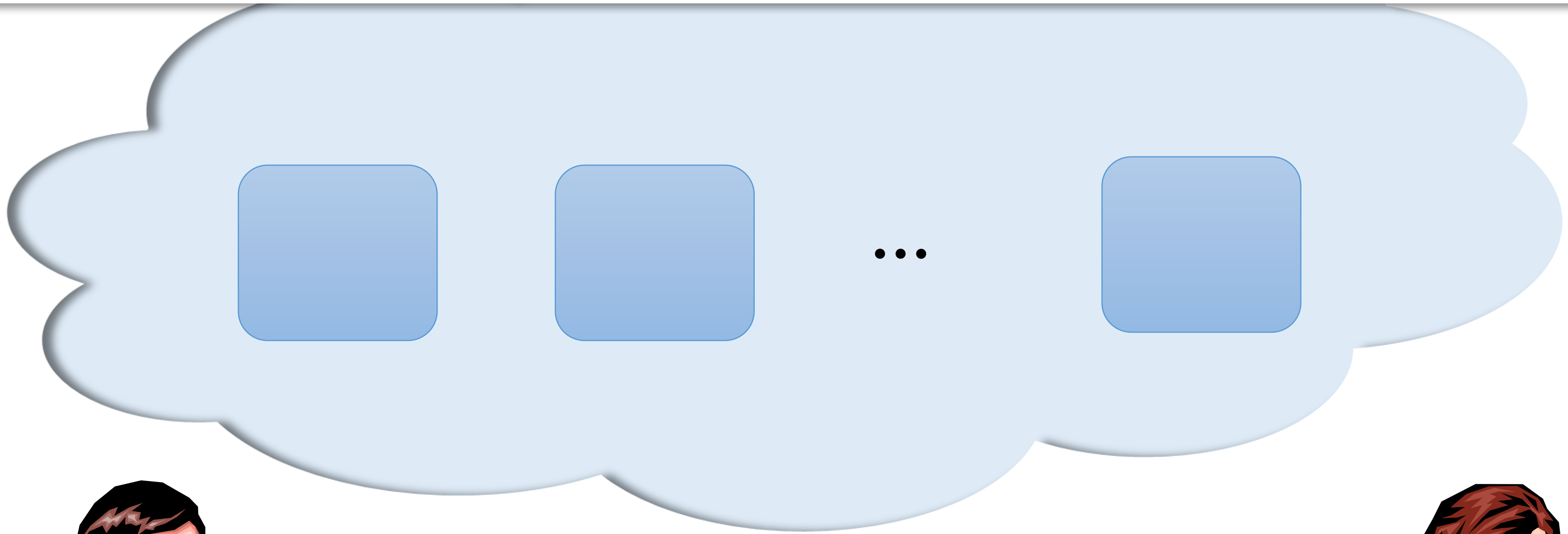
The Combined Protocol





1. Parties emulate the servers using passive 2PC
   a) Server's inputs are shared between the parties
2. Enforce honest majority among servers by "watching"
   a) Alice obtains Bob's shares for k-out-of-n servers
   b) Bob obtains Alice's shares for k-out-of-n servers

Active OLE from Passive OLE

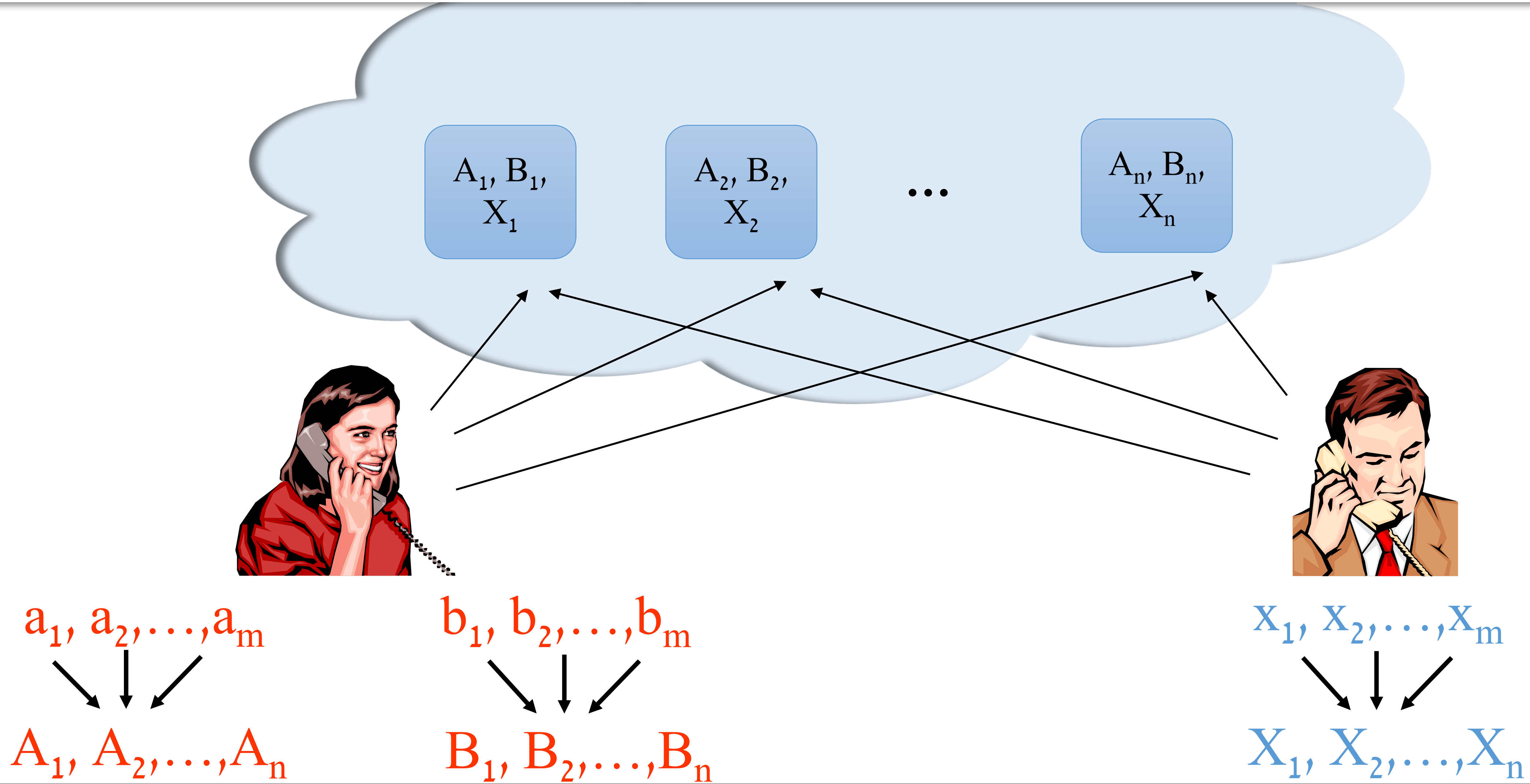


$a_1, a_2, \ldots, a_m$ → $A_1, A_2, \ldots, A_n$

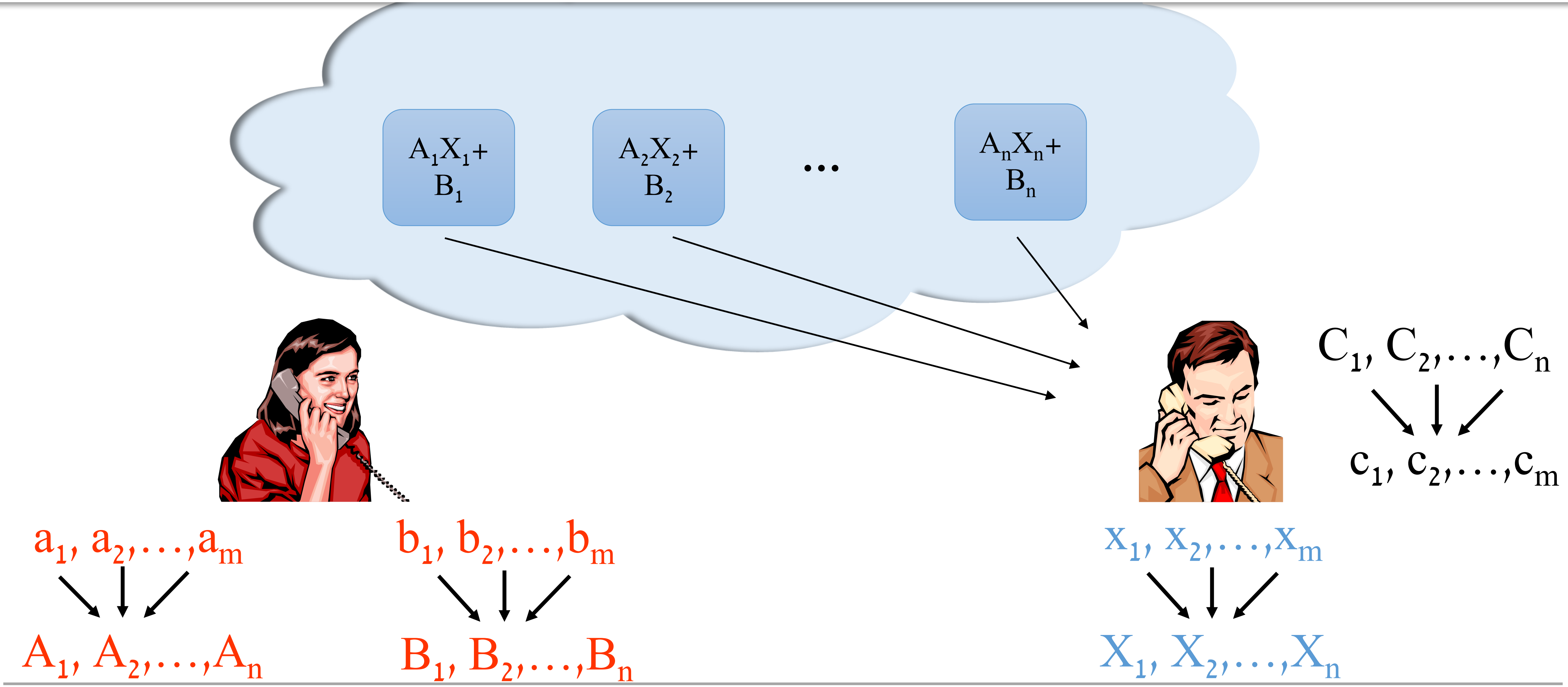
$b_1, b_2, \ldots, b_m$ → $B_1, B_2, \ldots, B_n$

$x_1, x_2, \ldots, x_m$ → $X_1, X_2, \ldots, X_n$

Active OLE from Passive OLE



Active OLE from Passive OLE



Black-Box Based on Any Passive OLE

1. More flexibility
   - Use any existing approach to passive OLE (e.g., lattice-based, group-based, code-based, etc.)
     - Does not need "ZK friendliness"
   - Off-the-shelf software/hardware implementation

2. Bonus feature

   "error-correct" weak implementations of passive OLE efficiently [in progress]
   - Constant correctness error
   - Constant privacy error

Summary

MPC-in-the-head is a useful tool for designing practical protocols

- Highly flexible and can be instantiated with different building blocks
- Optimized protocols achieve better parameters

- Much to explore in the context of concrete efficiency